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Analytic theory for foil impedance in spiral wound cell geometries

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HIGHLIGHTS

- Analytic solution for foil current and potentials in spiral wound cells with tabbed current collection.
- Analytic solution for cell impedance, and heat generation during discharge.
- Disentangle foil and stack impedance.
- Comparison of common and high efficiency tab arrangements.
- Worked examples for real world problems, aimed at non specialists.

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ABSTRACT

The analytic theory for 1D foil currents and potentials in spiral wound cells with arbitrary tab configurations is derived. The theory is extended to account for state of charge gradients along the length of the foils revealing how tab geometry can affect the slope of constant current voltage curves. Analytic expressions for cell level properties are derived for common tab configurations. The quality of many tab configurations is compared. A straight forward method is described for the common, previously unsolved problem of disentangling the foil and stack contributions of a measured cell impedance. Many other examples of practical problems and solutions are presented. The analytic theory shows how to directly calculate the foil geometry factor introduced in a previous article [1]. Simple methods are presented for adding corrections for foil effects to a stack centric cell model. The analytic results provide a lot of insight and ease of use that numerical solutions cannot easily deliver.

1. Introduction

Area Specific Impedance (ASI) is a useful concept for characterizing the rate or power capability of a particular cell chemistry [2–4]. This applies for both AC and DC impedance techniques. A long-standing problem with using ASI analysis for larger cell sizes is disentangling the impedance caused by the current collection system (tabs and foils) from the stack impedance. Nelson et al. [5] were able to avoid this problem by building cells with continuous current collection (current flows in the from edge along the full length of the foils), which reduces the effective impedance of the current collection system to near zero. However it is not easy to mass produce cylindrical cells with continuous current collection. Cells with a limited number of current collector tabs placed at strategic locations along the foil length represent the vast majority of commercial cells made today. It would be very useful to find a method for subtracting the effective foil impedance from the total measured cell impedance in order to obtain an accurate estimate of the stack ASI. We will show how this can be done for any tab configuration.

To our knowledge all previous attempts to model the effect foil current and potentials have focused on using numerical methods [6–13]. In this work we will show that to a very good approximation the problem of 1D foils with randomly positioned tabs can be solved analytically. It is well known that analytic results have a lot of advantages. Most notably, they provide much deeper understanding and they are much easier to use. Others [14–17] have also made progress with analytical solutions for discharge capacity, electrolyte salt concentration and solid phase Li concentration.

The field equations for current and voltage distribution along the length of the foils (x direction) have been described previously [7]. All foils are coated on 2 sides, which means that the standard commercial spiral wound cell is actually 2 cells in parallel. In this work we use the symbol $2i_s$ for the total stack current exiting a foil layer and the current entering one stack layer will then be i_s. We will use the symbol ρ_s to represent the area specific resistivity [Ω cm^2] of one stack layer, so the resistivity of 2 layers in parallel is

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½ρ_s. A more detailed discussion appears in Ref. [7]. The field equation for a region between tabs is:

$$\nabla_x^2 \phi_s(x) = -2W(R_A + R_C) i_s(x) \quad (1)$$

where $\phi_s(x)$ is the potential across the stack, $i_s(x)$ is the current density [mA cm^{-2}] running through one layer of the stack, W is the active width, $R_A = \rho_A/W_A t_A$ is the linear resistivity [Ω cm^{-1}] of the anode foil, ρ_A is the anode foil material resistivity [Ω cm], t_A is the anode foil thickness [cm], W_A is the anode foil width [cm], with similar definitions for the cathode. Following [7], the x direction goes along the length of the spiral wound foils, y direction goes through the stack, z direction corresponds to the height of the cell or the width of the electrode foils. Eq. (1) may look like a Poisson equation, but it is not because in the source term (RHS of Eq (1)), i_s and ϕ_s are intimately related. For example if one assumes a simple Ohmic relationship as in

$$\phi_s(x) = V_o(x) - \rho_s i_s(x) \quad (2)$$

where V_o is the open circuit potential [volt] across the stack (which should not be confused with the open circuit potential of the whole cell), then one obtains

$$\left(\nabla_x^2 - \alpha^2\right)\phi_s = -\alpha^2 V_o(x) \quad (3)$$

where

$$\alpha^2 = \frac{2W(R_A + R_C)}{\rho_s}. \quad (4)$$

A differential equation with the form of Eq. (3) is commonly referred to as a modified Helmholtz equation or a screened Poisson equation.

Solving for the foil potentials in a cell design with multiple tabs is a rather complex problem. In Ref. [7] we handled this by dividing the foil length into segments, with each segment bounded by a tab or the end of the foils. Multiple instances of (3) were numerically solved, separately for each segment. The boundary conditions for each segment are related to the external boundary conditions for the cell through a system of linear equations. Solving for the segment boundary conditions is not computationally expensive, but it is difficult to program correctly. Below we show how the general problem can be handled with just one segment, which should be easier to understand and program.

We have attempted to make this article somewhat comprehensive and provide a number of expressions for the various cell properties in tables and appendices. Casual readers can jump to the results section to see how tab configurations affect cell impedance, or to the Worked Examples section for step by step instructions on how to use this technology to solve real world problems.

2. Single segment theory

2.1. Reduced units

Before proceeding it will be useful to use dimensionless units in the x direction,

$$X = \frac{x}{L}$$

where L is the total active length in the x direction. We will also define $\lambda = \alpha L$. The reader is reminded that $(\partial^2/\partial x^2) = \nabla_x^2 = (1/L^2)(\partial^2/\partial X^2) = (1/L^2)\nabla_X^2$.

2.2. Derivation of field equations

The divergence of the foil current must account for current flow through the stack and current flow from the tabs. The stack portion is smooth, and the tabs portion can be approximated with Dirac delta functions [18], $\delta(x)$. This is approximate because the tabs and the welds between the tabs and foils will have finite width, but this width is very narrow in comparison to the length of the foils. Since the Dirac delta function is not commonly used in the electrochemistry literature it will be useful to review a few properties that will be required for understanding the developments below. One reason for developing the more complicated multisegment solution in Ref. [7] was fear of the delta function. All of the properties of the delta function required for this work are summarized in Table 1, and the delta functions and its first two integrals are plotted in Fig. 1. With Table 1 under our belt, fear of the delta function becomes unwarranted.

For a cell with an arbitrary number of tabs, the foil current divergences are

$$\nabla_X I_A(X) = -2LWi_s(X) + I\sum_{j\in A}^{n_A} f_j\delta(X - X_j) \quad (5)$$

$$\nabla_X I_C(X) = 2LWi_s(X) - I\sum_{j\in C}^{n_C} f_j\delta(X - X_j) \quad (6)$$

where I is the externally applied current, I_A and I_C are the currents along the anode and cathode foils, f_j is the current fraction flowing into tab j, and X_j is the location of tab j. $\sum_{j\in A}$ means that j indexes only over the anode tabs so that the anode and cathode each have a distinct set of js. For example $A = \{1,2,3,\ldots n_A\}$ and $C = \{n_A+1, n_A+2, n_A+3\ldots n_A+n_C\}$. Since these summations and their integrals will appear often we will define

$$\Sigma_C'' = \sum_{j\in C} f_j\delta(X - X_j) \quad (7)$$

$$\Sigma_A'' = \sum_{j\in A} f_j\delta(X - X_j). \quad (8)$$

$$\Sigma'' = r_A\Sigma_A'' + r_C\Sigma_C'' = \sum_j r_j f_j\delta(X - X_j) \quad (9)$$

where

$$r_j = \begin{cases} r_A = \dfrac{R_A}{R_A + R_C}, & j\in A \\ r_C = \dfrac{R_C}{R_A + R_C}, & j\in C \end{cases}$$

are the dimensionless partial foil resistances which will always satisfy $r_A + r_C = 1$. The indefinite integrals $\Sigma' = \int \Sigma'' dX$ and double integrals $\Sigma = \int \Sigma' dX$ are then easily determined from the relations in Table 1. The fractional currents obey the normalization conditions

$$\sum_{j\in C} f_j = \sum_{j\in A} f_j = 1$$

Using Ohms law we can convert (5) and (6) into expressions for foil potentials