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Static and fatigue failure of quasi-brittle materials at a V-notch using a Dugdale model

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ABSTRACT

The prediction of crack nucleation at stress concentration points in brittle and quasi-brittle materials may generally rely on either an Irwin-like criterion, involving a critical value of the generalized stress intensity factor of the singularity associated to the stress concentration, or on cohesive zone models. Leguillon's criterion enters the first category and combines an energy condition and a stress one. Thanks to matched asymptotics procedures, the associated numerical values at crack initiation under quasi-static monotonic loadings are shown to be comparable to those obtained using the Dugdale cohesive zone model. Both approaches are therefore adapted to the description of brittle and quasi-brittle fracture. A macroscopic Paris-like propagation law is derived from the Dugdale model through a relevant cumulating law at the microscopic scale of the process zone. Comparisons with experimental results are performed and display good agreement. The important matter of nucleation and growth of a fatigue crack at the root of a V-notch is finally addressed. A general Paris law featuring the elastic singularity exponent and then dependent on the V-notch angle can be expressed for small cyclic loadings in the early growth stage.

1. Introduction

The brittle fracture theory (Lawn, 1993) deals with the conditions of propagation of a pre-existing crack in brittle and quasi-brittle materials, but it is unable to address the emergence of a new crack at a stress concentration point like a V-notch root. Many efforts have been made to answer this question with a general criterion without going into the details of micromechanisms.

Two approaches can be highlighted, one goes through a non-local criterion and enters a general theoretical context while the other uses the cohesive zone models which have been developed and are more often used in a computational structures background. These two approaches share a common feature, they require two failure parameters to choose among the following three: the material toughness denoted either G_c (J m^{-2}) or K_{Ic} (MPa m$^{1/2}$), the tensile strength σ_c (MPa) and a characteristic length ℓ_c (m).

Within the non-local approach, a first family is based on a point-stress condition (McClintock, 1958; Leguillon, 2002; Leguillon and Yosibash, 2003; Taylor, 2008): failure occurs if the tensile stress exceeds a given critical value at a given distance of the V-notch root. Leguillon's criterion enters this family but the critical distance is no longer a material property, it depends on the local geometry. Its definition relies on an energy balance equation in addition to the maximum tensile stress condition (Leguillon, 2002). This approach is naturally extended to the average stress criterion: failure occurs if the tensile stress averaged on a given distance exceeds a given critical value (Novozhilov, 1969; Seweryn, 1994; Seweryn and Mroz, 1998; Seweryn and Lukaszewicz, 2002). The stress condition is generally transcribed into a condition on the generalized stress intensity factor (GSIF) characterizing the influence of the singular field associated with the V-notch, leading to an Irwin-like criterion. A second family is based on the strain energy density concept, failure occurs if the strain energy density (SED) exceeds a given value over a given volume (Sih, 1973; Yosibash et al., 2004) encompassing the stress concentration point. This second approach is generally less accurate.

After the pioneering works of Dugdale (1960) and Barenblatt (1962), the cohesive zone models (Tvergaard and Hutchinson, 1992; Planas and Elices, 1992, 1993; Xu and Needleman, 1994; Bazant and Planas, 1998) were originally developed for studying the fracture of interfaces in heterogeneous materials and particularly the mechanisms of delamination in composite laminates (Needleman, 1990; Allix and Ladeveze, 1992; Mi et al., 1998; Alfano and Crisfield, 2001). They model a process zone ahead of the crack tip or the stress concentration point. In this

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zone the material yields or damages but cohesive forces still act until the final fracture. Different profiles of the force/opening curve are proposed by authors (Alfano, 2006), they are characterized either by a peak stress and a critical opening or by a peak stress and a fracture energy corresponding to the surface located below the curve.

All these approaches are in general dedicated to quasi-static monotonic loadings; the literature becomes sparse when looking at the effects of fatigue loads in the vicinity of stress concentration points in quasi-brittle materials. Most of the papers agree to recognize that the GSIF is a relevant parameter at least to describe the appearance of short cracks, i.e. the life time at initiation (Taylor, 1999; Atzori et al., 2002, 2003; Lazzarin et al., 2003; Madi et al., 2004; Livieri and Lazzarin, 2005), although some others prefer using the SED (Lazzarin and Zambardi, 2001). The influence of the blunting of the V-notch is taken into account by a modified GSIF according to a parameter called the notch acuity (Boukharouba et al., 1995). Anyway these papers focus on different points like the fatigue strength presenting Kitagawa–Takahashi diagrams, the life time at initiation and the total life time assessments through S–N and Manson–Coffin curves, but none mentions the influence of the V-notch on a propagation law of Paris type.

In a broader context, we find the coupling between a cohesive law and a fatigue loading in different papers which share the same point of view. The damage irreversibility is obtained from a complementary mechanism to the cohesive law: a hysteresis due to different loading and unloading paths (Nguyen et al., 2001; Yang et al., 2001; Maiti and Geubelle, 2005; Bouvard et al., 2009; Ural et al., 2009).

In this work, Sections 3–6 are devoted to the comparison between the cohesive zone model of Dugdale and Leguillon's criterion for monotonic loading. They resume with more details the results established by Henninger et al. (2007). The two models give similar results for the prediction of crack nucleation at the root of a V-notch under monotonic loading. They are both well adapted to the description of the fracture of brittle or quasi-brittle materials. Furthermore, since the first model does not allow straightforward introduction of the concept of cumulative fatigue, it is naturally the second model which is used in the sequel to extend the results to the case of cyclic loadings. For such fatigue loads we exploit an idea proposed by Jaubert and Marigo (2006) and then used by Abdelmoula et al. (2009a,b). They suggest employing the opening cumulated during the cycles at a point instead of the instantaneous opening, and compare this parameter to the critical opening of the Dugdale law (Section 7). Clearly the two concepts coincide for a monotonic loading. In Section 8 we establish the fatigue law for a pre-existing crack in a simpler way than that proposed by Jaubert and Marigo. We observe that the selected cumulated law provides a poor agreement with experiments. Then a modified law is proposed in Section 9 and the resulting calibration is used to predict the onset of a fatigue crack at the root of a V-notch (Sections 10 and 11). Results strongly depend on the opening angle of the V-notch and can be written in the form of a Paris-like law featuring the elastic singularity exponent and then dependent on the V-notch angle during the early growth stage.

2. The Dugdale model

Preceding the pioneering work of Barenblatt (1962) on cohesive forces, Dugdale (1960) proposed a very simple law which can be considered in a way as a simplified model of plasticity prior to fracture. For a mode I pre-existing crack, it is assumed that the opening component $\delta[\![U_2]\!]$ (see Fig. 1 for the axis) of the

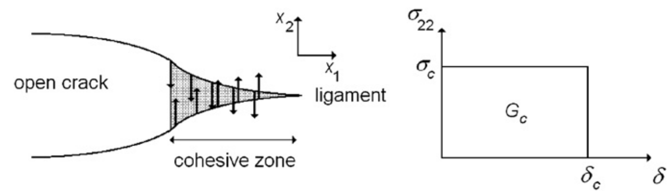

Fig. 1. The Dugdale cohesive zone model.

displacement jumps ahead of the crack tip and that a constant cohesive tension $\sigma_{22} = \sigma_c$ still acts if the opening does not exceed a given value δ_c (Fig. 1).

This cohesive force tends to close the zone and the cohesive zone length is such that no singularity (infinite values of the stress field) takes place at its end, the two faces come smoothly into contact (Fig. 1 left).

Dedicated to a crack, this model can be extended to any geometrical situation where a crack can nucleate due to stress concentrations. In quasi-brittle materials the cohesive peak force σ_c is chosen as the tensile strength of the material. In order to match the Griffith theory, the surface below the curve must equal the toughness G_c (Fig. 1). The result is a relationship between the three parameters (Bazant and Planas, 1998)

$$\delta_c = \frac{G_c}{\sigma_c} \tag{1}$$

In this paper we make a distinction between the incubation and the nucleation phases, especially for fatigue loadings. During the incubation phase, damage appears and the cohesive zone length can increase, i.e. the right end in Fig. 1 (left) moves, but the critical opening δ_c is not reached and the zone remains pinned at its left end. Nucleation takes place when the condition $\delta \geq \delta_c$ is fulfilled, i.e. when the left end starts to move.

3. Matched asymptotics and Dugdale zone

Let us consider a V-notched specimen loaded symmetrically so that fracture occurs along the bisector of the opening angle. We fit this line with a Dugdale cohesive zone and assume a priori that its length ℓ is much smaller than the dimensions of the specimen (the depth of the V-notch and the width of the remaining ligament in particular, Fig. 2).

The elastic solution $\underline{U}^{\ell}$ depends on this length which is unknown making the problem nonlinear.

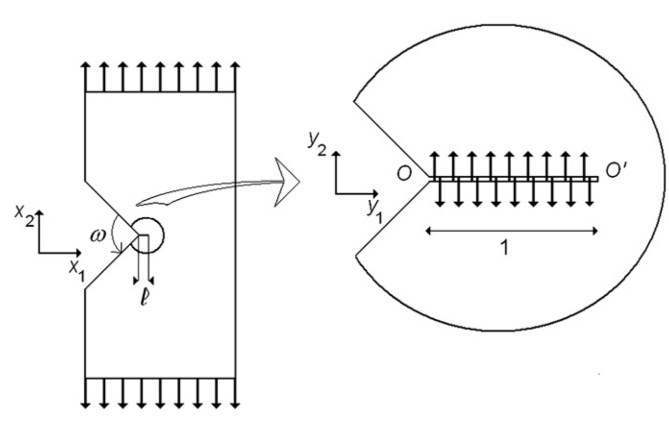

Fig. 2. The V-notched specimen and the Dugdale cohesive zone.