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A strain rate dependent yield criterion for isotropic polymers: Low to high rates of loading

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ABSTRACT

In this study, strain rate sensitivity of yield behavior in a semicrystalline polymer, Nylon 101, was experimentally investigated. A precise definition of yield was established for the polymer by deforming several specimens to certain levels of strain and measuring the residual strains after unloading and strain recovery. The material was then subjected to different loading conditions (uniaxial to multiaxial) at four different quasi-static and intermediate strain rates to determine several points on the material's yield loci. Due to positive strain rate sensitivity of this polymer, the material's yield loci expanded uniformly as the strain rates were increased to higher values. Further, an empirical hydrostatic pressure dependent yield equation (with four material constants) was developed to simulate these behaviors as a function of strain rate. The capability of the developed criterion was examined by simulating high strain rate yield behavior of the material in tension and in compression. The simulation results revealed very good correlations/predictions between the experimental data and the responses determined from the proposed yield criterion.

1. Introduction

It is well established that the yield behavior of polymeric materials subjected to external loads exhibits pronounced hydrostatic pressure dependency, as well as temperature and strain rate sensitivity (Anand et al., in press; Laiarinandrasana et al., in press; Ghorbel 2008; Rittel and Brill, 2008; Rittel and Dorogoy, 2008; Quang and He, 2008; Khan and Farrokh, 2006; Khan and Lopez-Pamies, 2002; Khan and Zhang, 2001; Caddell and Woodliff, 1980; Pae, 1977; Silano et al., 1974; Mears et al., 1969; Holliday et al., 1964). More efforts, however, have been made to investigate the effect of hydrostatic pressure on yield responses of these materials, both experimentally and theoretically. Holliday et al. (1964) were among the first investigators who experimentally showed that, unlike most metals, the yield strength of several amorphous and crystalline polymers increased with increase in the hydrostatic pressure. Further experimental results on different polymeric materials confirmed the earlier observations in the field (Mears et al., 1969; Pae, 1977), and inability of the existing yield criteria (e.g., von Mises and Tresca) to explain the observed different types of behavior. The first invariant of the stress tensor (i.e., the hydrostatic component) was incorporated into these criteria to involve the influence of hydrostatic pressure on the yield behavior (Ghorbel, 2008; Silano et al. 1974; Raghava et al., 1973; Bowden and Jukes, 1972; Raghava, 1972).

Bowden and Jukes (1972) proposed modified forms of the Tresca (Eq. (1)) and von Mises (Eq. (2)) criteria by linearly relating the maximum shear stress and the square root of the second deviatoric stress invariant, J_{2D}, to the first invariant of stress, J_1, respectively (Ghorbel, 2008):

$$\tau_{\mathrm{T}} = \tau^0_{\mathrm{MT}} - \frac{\mu_{\mathrm{MT}}}{3}J_1 \quad \text{with} \quad \tau_{\mathrm{T}} = \frac{1}{2}\sup(\sigma_i - \sigma_j) \tag{1}$$

where τ^0_{MT} and μ_{MT} are the material constants, while σ_i and σ_j represent principal stresses.

$$\tau_{\mathrm{Oct}} = \tau^0_{\mathrm{MMC}} - \frac{\mu_{\mathrm{MMC}}}{3}J_1 \quad \text{with} \quad \tau_{\mathrm{Oct}} = \frac{\sqrt{6J_{2D}}}{3} \tag{2}$$

where $J_{2D} = \frac{1}{2}S_{ij}S_{ji}$ and $S_{ij} = \sigma_{ij} - \frac{\sigma_{kk}}{3}\delta_{ij}$, while τ^0_{MMC} and μ_{MMC} are material constants. The described equations were applied to the experimental data on different polymers, and it was concluded that all polymers which deform relatively homogeneously (e.g., polyvinylchloride (PVC), epoxy resin, and high-density polyethylene (HDPE)) should obey a modified von Mises criterion; while those that deform inhomogeneously by the deformation of shear bands (e.g., polyethylene terephthalate (PET)) should follow a modified Tresca criterion (Bowden and Jukes, 1972).

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In 1973 Raghava et al. proposed a pressure modified von Mises yield criterion of the form:

$$3J_{2D} + (C - T)J_1 = CT \tag{3}$$

where C and T are the absolute value of the compressive and tensile yield strengths, respectively (Raghava et al., 1973). They used this equation to correlate the yield behavior of PVC, polycarbonate (PC), polymethylmethacrylate (PMMA), PVC, and polystyrene (PS). In general, good agreement was observed between the equation correlations and the experimental data.

Shortly after, Silano et al. (1974) proposed the following equation to account for hydrostatic pressure dependency of the yield strength in polymeric materials:

$$\sqrt{J_{2D}} = \sum_{i=0}^{N} \alpha_i (J_1)^i \tag{4}$$

where α_i are material constants. This equation is reduced to the von Mises and Drucker–Prager yield criteria when $N = 0$ and $N = 1$ respectively. Pae (1977) used this equation (Eq. (4)) with $N = 1$(i.e., $\sqrt{J_{2D}} = \alpha_0 + \alpha_1 J_1$, the Drucker–Prager criterion) and $N = 2$ (i.e., $\sqrt{J_{2D}} = \alpha_0 + \alpha_1 J_1 + \alpha_1 J_1^2$) to predict the yield behavior of polyoxymethylene (POM) and polypropylene (PP), respectively. They stated that the equation was the only criterion among others capable of predicting the yield behavior of these two polymers. A similar criterion was used by Khan et al. (1991) and applied to Berea sandstone; the experimental data at failure were used to formulate a failure curve in $\sqrt{J_{2D}} - J_1$ space incorporating a definite strength increase of the material with increase of the hydrostatic pressure.

Very recently, Ghorbel (2008) showed that the modified Tresca and von Mises yield criteria (Eq. (1) and Eq. (2)) were not capable of predicting the viscoplastic responses of all thermoplastic polymers when subject to any general biaxial stress state. To improve the prediction of isotropic polymers' yield behavior under any ranges of biaxial stress state, and to establish a generalized yield criterion which accounts for shear banding (when the main deformation mechanism is related to shear banding) along with hydrostatic pressure dependency, she proposed to include the third invariant of the deviatoric stress, J_{3D}, in the expression of the yield equation, f. The simplified equation was of the form:

$$f = \frac{3J_{2D}}{\sigma_{st}} \psi(J_{2D}, J_{3D}) + \frac{7(m-1)}{8} J_1 - \frac{7m}{8} \sigma_{st} \tag{5}$$

where;

$$\psi(J_{2D}, J_{3D}) = \left(1 - \frac{27}{32} \frac{J_{3D}^2}{J_{2D}^3}\right) \tag{6}$$

$m = \frac{\sigma_{sc}}{\sigma_{st}}$ while σ_{sc} and σ_{st} are the yield stresses in compression and in tension, respectively. The introduction of the third invariant of the deviatoric stress tensor in the proposed criterion did not affect the yield stress in uniaxial tension, uniaxial compression, or in purely hydrostatic pressure. However, differences in the calculated yield strength were observed under pure shearing. The application of this criterion to experimental data on PMMA, PC and PS showed some improvement over the yield loci drawn using other criteria (Ghorbel, 2008).

It is clear that, in contrast to numerous studies in the field concerning yield behavior of different polymeric materials, a yield criterion taking the strain rate sensitivity of these materials into consideration does not exist (to the best of the authors' knowledge). In this study, an effort has been made to experimentally investigate the yield behavior of a semicrystalline polymer, Nylon 101, under different loading conditions, at different quasi-static and intermediate strain rates. In addition, a yield criterion is introduced to predict the yield behavior of different polymeric materials as a function of strain rate (low to high rates of loading).

2. Experimental procedure

The material used in this study, Nylon 101, was obtained from Total Plastics Inc. ™. All specimens were prepared from 25.4 mm diameter rods, and from the same manufacturing batch. Different specimens with different designs and geometries were prepared to conduct experiments under different loading conditions. Fig. 1 shows the geometries and the dimensions of the samples used to perform all the experiments in this work. All experiments were carried out using an MTS 809 Axial/Torsional Material Testing System. The experiments included:

2.1. Uniaxial tension experiments

The specimens (Fig. 1A) were prepared according to the ASTM E-8 so that the axes of the solid dog-bone samples were parallel to the rod's extrusion direction. The experiments were conducted at four different strain rates of 10^{-5}, 10^{-4}, 10^{-2} and 1 s^{-1}. The strains were measured by taking advantage of high elongation strain gages (KFEL-2-120-C1, Kyowa Ltd.), and an axial extensometer (see Fig. 2A).

2.2. Uniaxial compression experiments in direction 1

The cylindrical solid specimens (L = 2.3 cm and D = 1.9 cm) were prepared so that the axes of specimens were parallel to the rod's extrusion direction (Fig. 1B). These experiments were also conducted at different strain rates of 10^{-5}, 10^{-4}, 10^{-2} and 1 s^{-1}. The strains were measured by using high elongation strain gages (KFEL-2-120-C1, Kyowa Ltd.) as shown in Fig. 2B.

Different approaches have been adopted for identifying the onset of irreversible deformation in polymeric materials. For a detailed description of such approaches the reader is referred to Ishai and Bodner (1970); Raghava et al. (1973); and Ghorbel (2008). Raghava (1972) indicated that a value between 0.3% and 0.9% off-set definition of yield can be used to determine yielding in most

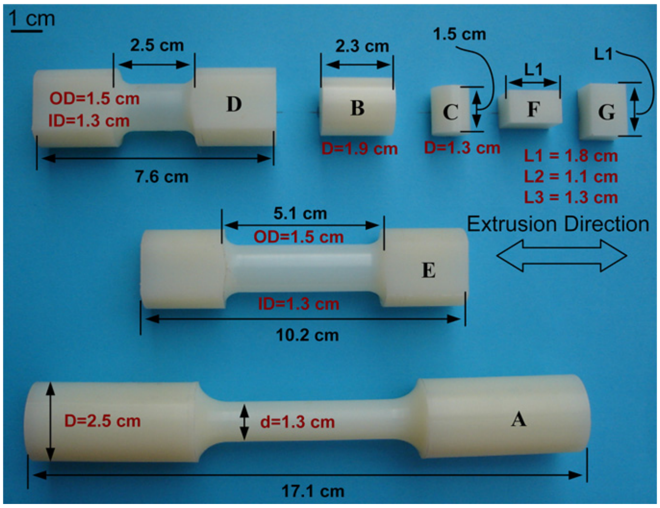

Fig. 1. The designs and geometries of the specimens used to perform all the experiments in this study.