European Journal of Mechanics A/Solids 56 (2016) 19–30

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

A micro-mechanics based strain gradient damage model: Formulation and solution for the torsion of a cylindrical bar

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ARTICLE INFO

Article history:
Received 30 December 2014
Accepted 6 October 2015
Available online 28 October 2015

Keywords:
Micro-mechanics
Damage model
Strain gradient

ABSTRACT

The present paper is devoted to the proposal of a theoretical formulation for an isotropic damage model with strain gradient. The approach is based on the non-local estimates of Drugan and Willis (1996), exposed in terms of energetic methods, for the purpose of damage modelling. We first focus on the derivation of the non-local constitutive equations for the damage model which is fully analysed from the thermodynamics point of view. It is shown that the positive definiteness of the thermodynamic potential and the intrinsic dissipation are not ensured for every loading paths. The choice of the damage variable is briefly discussed. The equilibrium equation and the boundary conditions are presented. Then, the model is applied to the study of strain gradient torsion problem, for which the axi-symmetric solution is established. This allows us to study the size effect and to evaluate the impact of the non-local term on the damage evolution and the non-linear behaviour of the bar.

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1. Introduction

In the context of durability studies of civil engineering buildings, damage mechanics offer an interesting framework to model the irreversible deterioration of quasi-brittle materials. However, when softening occurs, strain localisation can be observed and local damage theory becomes unreliable. From the numerical point of view, the loss of objectivity implies a dependency between the amount of dissipated energy and the mesh refinement. Non-local damage models are widely used to overcome those issues and can be split into three different categories: regularisation of the strain variable as in the integral non-local model (Pijaudier-Cabot and Bažant, 1987) or in the implicit gradient model (Peerlings et al., 1996; Kuhl and Ramm, 2000), introduction of the damage gradient in the strain energy (Frémond and Nedjar, 1996; Lorentz and Andrieux, 1999; Pham and Marigo, 2013) or introduction of the strain gradient in the strain energy (Zhou et al., 2002).

Another important motivation of nonlocality is the need to capture size effects observed in fracture experiments on concrete (Walsh, 1972). Indeed, the size of the fracture process zone is independent of the size of the structure – provided it does not interfere with its boundaries – and therefore it can be observed that the nominal strength of geometrically similar specimens is dependent on the structure size. A characteristic length, related to the size of the fracture process zone, is needed to describe the transitional type of size effect. A review on nonlocality, including an historical summary of its main motivations can be found in (Bazant and Jirasek, 2002).

These regularised models introduce additional material parameters whose calibration can be difficult to carry out. Indeed, the identification of these material parameters cannot be achieved through simple experiments due to the localisation of the fields, which generally introduce structural effects in the mechanical response. Moreover, most of the time the additional parameters are independent of the level of damage or stress – a proposal can be found for the non-local integral regularisation method where the internal length depends on the stress level (Giry et al., 2011) – even though the non-local interactions are expected to change according to the state of the medium.

A key question concerning non-local damage models of quasi-brittle materials is that of the physical origin of the non-locality and its proper incorporation in the continuum damage mechanics framework. This question has been earlier pointed out in several papers among which (Bazant, 1994) for the non-local integral regularisation, (Andrieux et al., 1996) for the regularisation with a damage gradient and (Li, 2011) for the regularisation with a strain gradient in 2D.

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http://dx.doi.org/10.1016/j.euromechsol.2015.10.001
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An appropriate way to establish physical-based regularised damage models – and then to circumvent the aforementioned problems of the non-local material parameters – is to derive the constitutive law by means of homogenisation techniques. Therefore, the main objective of the present paper is to formulate and study a micro-mechanics based non-local damage model with strain gradient from the non-local estimates of Drugan and Willis (1996). The relevance of such a model, to which numerous works refer in the litterature, and its ability to capture the size effects due to nonlocality is investigated in this work.

The paper is organised as follows. First, from the non-local micro-mechanical analysis of Drugan and Willis, we derive the basic elements of the proposed macroscopic non-local damage model. The thermodynamic potential and the state laws are presented, as well as the damage criterion and the evolution laws. A careful use of the model is done in order to guarantee the positiveness of the thermodynamic potential and of the intrinsic dissipation. Then, we present the equilibrium equation and the boundary conditions which are classically deduced from the principle of virtual work. Finally, we establish the exact solution for the torsion of a cylindrical bar which obeys the non-local law. Size effects due to the non-local behaviour are illustrated. The impact of the non-local term on the damage evolution and the non-linear behaviour of the bar is also evaluated.

2. Formulation of a micro-mechanics based strain gradient damage model

2.1. Drugan and Willis non-local elasticity model (1996)

The present study is based on the micro-mechanical non-local constitutive equations established by Drugan and Willis (1996). Considering a class of two-phase composites with an isotropic and statistically uniform distribution of phases, these authors showed that the macroscopic strain energy depends on both the macroscopic strain state E and the macroscopic strain gradient state ∇E:

$$w(E,\nabla E)=\frac{1}{2}E\odot_2{}^{(4)}\mathbb{C}^{hom}\odot_2E+\frac{1}{2}\nabla E\odot_3{}^{(6)}\mathbb{B}^{hom}\odot_3\nabla E \tag{1}$$

where $\odot_n$ is the nth tensor contraction between a tensor $\mathbb{A}$ of order greater than n and a tensor a of order n such that $[\mathbb{A}\odot_n a]_{i,\ldots,j}=\mathbb{A}_{i,\ldots,jk,\ldots,l}a_{k,\ldots,l}$. For clarification, the energy potential given in equation (1) can be rewritten in indicial notation: $w(E_{ij},E_{ij,k})=\frac{1}{2}E_{ij}{}^{(4)}\mathbb{C}^{hom}_{ijpq}E_{pq}+\frac{1}{2}E_{ij,k}{}^{(6)}\mathbb{B}^{hom}_{ijkpqr}E_{pq,r}$.

The energy potential (1) is continuously differentiable with respect to the macroscopic strain tensor E and the macroscopic strain gradient tensor ∇E. State laws can therefore be derived, thus providing the macroscopic stress tensor Σ and the macroscopic double stress tensor T:

$$\Sigma=\frac{\partial w}{\partial E}={}^{(4)}\mathbb{C}^{hom}\odot_2E;\quad T=\frac{\partial w}{\partial\nabla E}={}^{(6)}\mathbb{B}^{hom}\odot_3\nabla E \tag{2}$$

If the structure is made of an isotropic matrix reinforced or weakened by a uniform dispersion of non-overlapping identical isotropic spheres, both the 4th order tensor ${}^{(4)}\mathbb{C}^{hom}$ and the 6th order tensor ${}^{(6)}\mathbb{B}^{hom}$ can be explicitly constructed.

Let us consider a linear elastic matrix whose properties are the shear modulus μ_0 and the bulk modulus κ_0. This matrix contains an uniform dispersion of non-overlapping identical spherical voids of volume concentration c defined by:

$$c=\frac{4}{3}N\pi a^3 \tag{3}$$

where a and N respectively stand for the voids radius and the voids density i.e. the number of voids per unit of volume.

The classical Hashin-Shtrikhman upper bound provides the macroscopic stiffness tensor of the porous material:

$${}^{(4)}\mathbb{C}^{hom}=3\kappa^{hom}{}^{(4)}\mathbb{J}+2\mu^{hom}{}^{(4)}\mathbb{K} \tag{4}$$

where ${}^{(4)}\mathbb{J}=I\otimes I/3$ and ${}^{(4)}\mathbb{K}={}^{(4)}\mathbb{I}-{}^{(4)}\mathbb{J}$ are the two isotropic 4th order projectors, ${}^{(4)}\mathbb{I}=I\otimes_s I$ is the symmetric 4th order unit tensor, I is the second order unit tensor, $\otimes$ is the tensor product and $\otimes_s$ the symmetric tensor product. The homogenised shear modulus and bulk modulus are respectively given by:

$$\mu^{hom}=\mu_0\frac{(1-c)(9\kappa_0+8\mu_0)}{9\kappa_0+8\mu_0+6c(\kappa_0+2\mu_0)} \tag{5}$$

$$\kappa^{hom}=\kappa_0\frac{4(1-c)\mu_0}{3c\kappa_0+4\mu_0} \tag{6}$$

The isotropic 6th order tensor obtained by Drugan and Willis reads:

$$\begin{aligned}{}^{(6)}\mathbb{B}^{hom}=&-\beta^{hom}\left({}^{(6)}\mathbb{K}_2+\frac{7}{4}{}^{(6)}\mathbb{K}_4-\frac{7}{2}{}^{(6)}\mathbb{K}_6\right)\\&-\left(\frac{3}{4}\gamma^{hom}+\frac{11}{4}\beta^{hom}\right)\left({}^{(6)}\mathbb{J}_1+{}^{(6)}\mathbb{J}_2+{}^{(6)}\mathbb{J}_4+{}^{(6)}\mathbb{J}_5\right)\\&-\left(\frac{9}{4}\gamma^{hom}+3\beta^{hom}\right)\left({}^{(6)}\mathbb{J}_3+{}^{(6)}\mathbb{J}_6\right)\\&-\left(2\gamma^{hom}+\frac{3}{2}\beta^{hom}\right)\left({}^{(6)}\mathbb{J}_7+{}^{(6)}\mathbb{J}_8\right)\\&-\left(\frac{3}{2}\gamma^{hom}+2\beta^{hom}\right){}^{(6)}\mathbb{J}_9\end{aligned} \tag{7}$$

where the 6th order tensors ${}^{(6)}\mathbb{K}_n$ for $n=1,\ldots,6$ and ${}^{(6)}\mathbb{J}_m$ for $m=1,\ldots,9$ constitute an irreducible basis for the 6th isotropic order tensors as proposed by (Monchiet and Bonnet, 2011). Their expressions are given in Appendix A.

The non-local parameters β^{hom} and γ^{hom} depend on the porosity c and the voids radius a and are given by:

$$\gamma^{hom}=4ca^2\frac{(2-c)(1-c)^2}{5(1+2c)}\mu_0^2(3\kappa_0+4\mu_0)\times\frac{5(3\kappa_0+4\mu_0)[21\kappa_0\mu_0-2\mu_0(3\kappa_0+8\mu_0)]-12(1-c)\kappa_0\mu_0(3\kappa_0+\mu_0)}{21(3c\kappa_0+4\mu_0)[5\mu_0(3\kappa_0+4\mu_0)-6(1-c)\mu_0(\kappa_0+2\mu_0)]^2} \tag{8}$$

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