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# Effects of inter-fibre spacing on damage evolution in unidirectional (UD) fibre-reinforced composites

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ABSTRACT

A three dimensional (3D) micromechanical study has been performed in order to investigate local damage in UD composite materials under transverse and longitudinal tensile loading. In particular, the influence of non-uniform distribution of fibres in RVEs (representative volume element) with a hexagonal packing array and the effects of thermal residual stresses has been investigated. To examine the effect of inter-fibre spacing and residual stress on failure, a study based on the Maximum Principal Stress failure criterion and a stiffness degradation technique has been used for damage analysis of the unit cell subjected to mechanical loading. Results indicate a strong dependence of damage onset and its evolution from the fibres position within the RVE. Predicted mechanical properties, damage initiation and evolution are also clearly influenced by the presence of residual stress.



## 1. Introduction

In a unidirectional composite at micro level fibres are embedded in the matrix material parallel to the longitudinal direction. In the transverse cross-section a random distribution of fibres exists resulting in transverse isotropic properties. To model the fibre distribution in the transverse plane, the simplest idealisation for analytical or numerical approaches is to adopt for example a hexagonal or a square array of fibre packing geometry. These packing geometries assume a uniform distribution and periodic packing of the fibres and are greatly convenient in numerical modelling since representative elements of a limited size can be used. Moreover the boundary conditions for both analytical and numerical analyses can be constructed in a simple way (Sun and Vaidya, 1996). As in UD composites fibres arrangement in the matrix over the transverse cross section is generally at random, a more realistic response on a local as well as a global level can be obtained by taking into account a non-uniform packing geometry. Boundary conditions, in micromechanical finite element analyses of UD composites with non-uniform distribution of fibres, can no longer be prescribed precisely due to the lack of symmetry. Several authors (Wongsto and Li, 2005; Bulsara et al., 1999; Aghdam and Dezhsetan, 2005; Gusev et al., 2000; Rossoll et al., 2005; Knight et al., 2003) have applied numerical method to investigate the effects of non-uniform fibre distribution on the overall behaviour of composites at their microscale. The geometrical structures examined are in general based on a multi-fibre RVE approach in which the RVEs are formed either by uniformly spaced fibre arrangements (square and hexagonal array) or randomly placed fibres within the matrix. In particular, Aghdam and Dezhsetan (2005) extended the geometry of the Simplified Unit Cell (SUC) model, to study effects of random fibre arrangement on the mechanical and thermal characterisation of unidirectional composites to predict the behaviour of a fibrous composite subjected to thermal and mechanical, normal and shear, loading. Gusev et al. (2000) used a combination of numerical and experimental techniques to study the fibre packing and elastic properties of a transversely random unidirectional glass/epoxy composite. The authors showed that measured and numerical results were in excellent agreement and moreover randomness of the composite microstructure had a significant influence on the transverse composite elastic constants while the effect of fibre diameter distribution was small and unimportant. A comparison between analytical approaches and finite element analyses (FEA) for varying fibre distributions, ranging from single fibre unit cells to complex cells was performed by Rossoll et al. (2005) on metal matrix composites. Analysis of micro-fields showed that the main cause for deviation from the equistrain rule of mixtures is a stiffening effect of matrix confinement when surrounded by touching fibres arranged as "rings". The Boundary Element Technique (BEM) and the embedded cell approach (ECA) were adopted by Knight et al. (2003) to investigate the micromechanical response of fibre-reinforced materials. Non-periodic arrangements give rise to higher local stresses, and the magnitudes of these stress concentrations have a strong dependence on the ligament length (distance between the two neighbouring fibres that have a common high-stress region), and to a lesser extent on the angle relative to the applied load (angle between a plane con-

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taining the two fibre centres and the applied load). The use of simple cells accounting for inhomogeneous fibre distribution but still able to preserve the convenience of RVE when applying the boundary conditions have been adopted by Zhang et al. (1998a, 1998b) in order to perform micromechanical investigation of the transverse creep behaviour of UD reinforced glass–fibre composites with an unsaturated polyester as matrix that shows a non-linear viscoelastic constitutive behaviour. The new geometries seem to be more suitable than the traditionally used regular packing geometries. In fact they displayed good agreement with experimental data and moreover reveal a detailed stress and strain distribution and redistribution over time on a local level. Most of the investigations described above are based on multi-fibre RVEs and are unable to describe accurately and locally damage onset and its evolution that usually requires a fine mesh refinement around the fibre/matrix interface. In addition, they account rarely for the effect of residual stresses arising during the curing process. In fact the local stress distributions after chemical shrinkage and the following cooling could influences the onset of microcracks in the matrix especially in presence of a random distribution of the fibre over the cross-section (Gentz et al., 2004). Residual stresses have important effects on the thermo-mechanical behaviour of composite materials and, moreover, the resulting stresses are sufficient to initiate fracture within the matrix immediately around the fibre (Gentz et al., 2004). In Zhao et al. (2007) and Maligno et al. (2008) effects of residual stress on uniform square and hexagonal unit cells were broadly object of study by mean a numerical approach. Nevertheless results from these investigations showed beneficial and detrimental effects of residual stress depending on different variables (e.g. fibre content, matrix ultimate strength) never failure was detected during chemical shrinkage or cooling process as it is sometimes likely to occur in a real curing process. The present study aims to evaluate the local response of non-uniform hexagonal unit cells that undergo residual stresses. In the present work, 3D finite element analysis has been used to study the overall response of a simple unit cell model with a hexagonal array of the fibres under transverse and longitudinal uniaxial loadings in which a "non-symmetric" distribution of fibres, by keeping the fibre volume fraction constant, modify locally damage onset and its evolution. Effects of residual stresses arising during the curing process on non-symmetric unit cells have been also investigated.

## 2. Finite element modelling

### 2.1. Micromechanical model

Composite materials properties, e.g. strength and stiffness, are dependent upon the fibre volume fraction and individual properties of the constituent fibre and matrix materials and the estimation of damage and failure progression is complex if compared to that of conventional metallic materials. In the micromechanical approach, the constituent fibre and matrix materials and their interaction are distinctively considered to predict the overall behaviour of the composite material structure. The advantage of the micromechanical model is that the stresses can be associated and related to each constituent (fibre and matrix). Therefore, failure can be identified in each of these constituents and the appropriate property degradation can be modelled. Here, the micromechanical model considers a unit cell in which fibre and matrix are assumed to be perfectly bonded to the fibres throughout the analysis, with fibres arranged in a hexagonal cross section array (Sun and Vaidya, 1996) by assuming the repetitive or periodic nature of the fibre and matrix materials. It should be noted that the present study deals with the microstructure of composite materials and in particular with the fibre distribution within the *tow*. Therefore, it is likely that high concentration of fibres (e.g. 70%) can be a reached, especially in ideal hexagonal packing arrays. For that reason 3D unit cells with a fibre content of 70% has been used in these investigations. The RVE considered for this study is displayed in Fig. 1. The meshes generated for the micro-models investigated are based either on quadratic hexahedral or quadratic wedge elements. The number of elements varies approximately from 15 500 to 51 000 depending on the element type, mesh refinement at fibre/matrix interface and fibre packing geometry. In particular high number of elements has been used in the meshes with wedge elements to achieve the same elastic response of hexahedron based meshes. Mesh sensitivity analysis suggests that the meshes of about 15 000 elements are fine enough to produce accurate results compared to a mesh with elements multiplied for a factor 1.5, 2 and 2.5, with a difference within 0.2%–0.3% in terms of residual stress and failure strain level but they lose the capacity to describe accurately the damage evolution within the matrix. Furthermore, the number of layers in the longitudinal ($x$-direction) used in the 3D models varies from three to five. Hence, the micromodels, in general, contain not less than 20 000 elements.

**Table 1**
Mechanical and thermal properties of fibre and matrix.

| Material properties | E-Glass | Epoxy |
|---|---|---|
| Longitudinal modulus, $E_1$ (GPa) | 80 | 3.35 |
| Transverse modulus, $E_2$ (GPa) | 80 | 3.35 |
| Poisson's ratio, $\nu$ | 0.2 | 0.35 |
| Shear modulus, $G$ (GPa) | 33.33 | 1.24 |
| Longitudinal tensile strength, $X_T$ (MPa) | 2150 | 80 |
| Longitudinal compressive strength, $X_C$ (MPa) | 1450 | 120 |
| Longitudinal tensile failure strain, $Y_T$ (%) | 2.687 | 5 |
| Longitudinal compressive failure strain, $Y_C$ (%) | 1.813 | – |
| Shear strength, $S$ (MPa) | 1200 | 70 |
| Thermal coefficient, $\alpha$ ($10^{-6}$ °C$^{-1}$) | 4.9 | 58 |

### 2.2. Material

The constituent materials used in this investigation are glass fibre and epoxy resin, whose properties are given in Table 1. The properties of glass fibre are assumed to remain constant and independent of the temperature change. However, for the epoxy resin, thermal transition temperatures such as the glass transition temperature $T_g$ strongly affect mechanical properties. In order to represent this behaviour accurately the material properties of the resin are defined as a function of temperature. The following relations are used:

(a) Poisson's ratio is assumed to be temperature independent.
(b) To evaluate the variation of Young's modulus $E$ over the temperature range from curing to room temperature, the total temperature range has been divided into three regions:
- $T_g - \Delta T \leqslant T \leqslant T_g + \Delta T$, in which $E$ varies greatly.
- $T > T_g + \Delta T$, the matrix is in liquid or rubbery state and $E$ has a very small value.
- $T < T_g - \Delta T$, the matrix is in solid state and $E$ changes only slightly.

For each region, the modulus is obtained using the following functions Zhang et al. (1998a, 1998b),

$$E(T) = E(T_r)\exp\left(-k_1\frac{T - T_r}{T_g - \Delta T - T_r}\right), \quad T < T_g - \Delta T; \tag{1}$$

$$E(T) = E(T_g - \Delta T)\exp\left(-k_2\frac{T - T_g + \Delta T}{\Delta T + \Delta T}\right), \quad T_g - \Delta T \leqslant T \leqslant T_g + \Delta T; \tag{2}$$

$$E(T) = 0.01E(T_r), \quad T > T_g + \Delta T; \tag{3}$$

with: