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Research on the process of micro-crack damage evolution and coalescence in brittle materials

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ABSTRACT

Damage and failure of quasi-brittle materials are caused by evolution and coalescence of micro-cracks. To solve the problem of elliptical micro-crack growth at the elastic deformation stage, a method of complex potential functions is proposed and the effect of the initial orientation on micro-crack growth and deflection is discussed. The critical stress condition for the initial damage is derived according to the criterion of micro-crack growth. Based on energy conservation during wing-crack propagation, a damage constitutive model is developed with the strain criterion created in the condition of micro-crack coalescence. The stress–strain curves of quasi-brittle materials in uniaxial compression obtained based on this model are examined with the experimental results. In conclusion, (1) wing crack growth, propagation, and coalescence at the tips of micro-cracks eventually lead to the formation of an anisotropic effective compliance tensor in the damaged material; (2) a large number of micro-crack coalescence is a highly nonlinear phenomenon resulting in failure of materials; and (3) the effective elastic modulus of damaged material decreases with the wing crack propagation length increasing.

1. Introduction

The elastic modulus of a material gradually decreases with increase in the number and size of micro-cracks in it during its evolution. When the number and size of micro-cracks reach a certain stage, the interactions among the cracks become very important. For example, the coalescence between cracks is a strong or nonlinear interaction occurring in the material during its fracture and failure. To obtain the strength limit under which the material fails, the nonlinear condition of coalescence between the micro-cracks must be studied.

Currently, a lot of researches focus on the growth and interaction of micro-cracks. Micromechanical damage models for brittle materials have been developed based on the mechanism of micro-crack propagation [1–7]. When subjected to tensile loading, the micro-cracks whose normal vectors approach to the stress direction of the maximum principle will propagate first. As a rule, the fracture surface is perpendicular to the maximum tensile stress direction. However, the evolution of micro-cracks in compression becomes more complicated. Some experimental and theoretical researches have shown that micro-cracks develop in different ways, such as becoming closed, frictional sliding, intergranular propagating and kink propagating [8,9]. Damage evolution of materials by the method of the domain of micro-crack growth (DMG) also have been studied [10,11]. Two methods, shielding and increasing stress intensity factor, have been proposed and used to deal with the interaction between micro-cracks [12–18].

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Calculating the propagation of micro-cracks depends on their structural characteristics, and changes in which during elastic deformation cause their propagation to be affected during nonlinear deformation. Therefore, the evolution of micro-cracks during elastic deformation should be taken into consideration before their propagation and coalescence. The coalescence is the direct cause of the material's fracture. The critical propagation length of micro-cracks in many damage models [19] is taken as a control parameter of fracture criterion and its value calculated by the condition of coalescence is of physical significance.

2. Growth of elliptical micro-crack

Fig. 1 shows schematic of the stress state of a representative unit containing an elliptical micro-crack. The representative unit is subjected to a far-field loading $\tilde{\sigma} = \sigma_1 \bar{e}_{x'}\bar{e}_{x'} + \sigma_2 \bar{e}_{y'}\bar{e}_{y'}$. The length of the elliptical micro-crack is $2a_0$ in its long axis and $2b_0$ in its short axis, where $b_0 = \rho a_0$ and $0 < \rho < 1$. Its orientation is at an angle of β with the direction of loading σ_1.

To study the growth of the elliptical micro-crack in the continuum medium subjected to tensile stress, for simplicity, the local coordinate system is converted into the elliptic coordinate system with the following relations:

$$\begin{cases} x = c \ \cosh \ \eta \ \cos \ \theta, \theta \in [0, 2\pi) \\ y = c \ \sinh \ \eta \ \sin \ \theta, \eta \in [\eta_0, +\infty) \end{cases} \tag{1}$$

where $c = \frac{a_0}{\cosh \ \eta_0} = \frac{b_0}{\sinh \ \eta_0}$, the boundary of the elliptical micro-crack is given by $\eta = \eta_0$, $\eta_0 = \operatorname{arctanh} \rho$. $\eta \to +\infty$ corresponds to $x^2 + y^2 \to +\infty$. Assuming that $\xi = \eta + i\theta$, then $z = c \cosh \xi$.

The boundary value of elliptical micro-crack subject to plan stress in an elastic continuum is solved by the method of complex potential function. The stress and displacement fields satisfying Eq. (2) are the solutions,

$$\begin{cases} \sigma_x + \sigma_y = 4 \ \mathrm{Re} \ \psi'(z) \\ \sigma_y - \sigma_x + 2i\tau_{xy} = 2[\bar{z}\psi''(z) + \chi''(z)] \\ 2G_0(u_x + iu_y) = \left[\frac{3-\upsilon_0}{1+\upsilon_0}\bar{\psi}(\bar{z}) - \bar{z}\psi'(z) - \chi'(z)\right] \\ 4 \ \mathrm{Re} \ \psi'(z) = \sigma \quad x^2 + y^2 \to \infty \\ 2[\bar{z}\psi''(z) + \chi''(z)] = -\sigma \quad x^2 + y^2 \to \infty \end{cases} \tag{2}$$

where $\psi(z)$ and $\chi(z)$ are complex functions.

$$(u_x + iu_y) = \frac{c}{8G_0}(\sigma_1 + \sigma_2)f_1(\xi) + \frac{c}{8G_0}(\sigma_1 - \sigma_2)[f_2(\xi) + f_3(\xi) + f_4(\xi)] \tag{3}$$

where

$$\begin{aligned} f_1(\xi) &= \tfrac{3-\upsilon_0}{1+\upsilon_0} \sinh \ \xi - |\cosh \ \xi|^2 \tfrac{1}{\sinh \ \xi} - \tfrac{\cosh \ 2\eta_0}{\sinh \ \xi} \\ f_2(\xi) &= \tfrac{3-\upsilon_0}{1+\upsilon_0}(e^{2\eta_0} \cos \ 2\beta \ \cosh \ \xi - e^{2\eta_0 - 2i\beta} \ \sinh \ \xi) \\ f_3(\xi) &= -\cosh \ \bar{\xi}(e^{2\eta_0} \ \cos \ 2\beta - e^{2\eta_0 + 2i\beta} \ \coth \ \xi) \\ f_4(\xi) &= -\left[\tfrac{\cos \ 2\beta}{\sinh \ \xi} + e^{2\eta_0} \tfrac{\sinh \ 2(\eta_0 + i\beta - \xi)}{\sinh \ \xi}\right] \end{aligned}$$

Let $\xi = \eta_0$, and the displacement at the tip of the long axis of the elliptical micro-crack is obtained

$$u_{ax} = \frac{c}{8G_0}[(\sigma_1 + \sigma_2)g_1 + (\sigma_1 - \sigma_2)g_2] \tag{4}$$

$$u_{ay} = \frac{c}{8G_0}(\sigma_1 - \sigma_2)g_3 \tag{5}$$

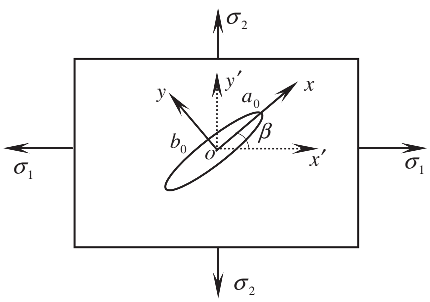

Fig. 1. Schematic of an elliptical micro-crack.