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Mechanical analysis on extensional and flexural deformations of a thermo-piezoelectric crystal beam with rectangular cross section

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ABSTRACT

One-dimensional equations of a thermo-piezoelectric beam are obtained by using of double power series expansion along the thickness and width directions from three-dimensional equations of the linear theory of piezoelectricity. Both the extension and flexure as well as shear deformations are considered, accompanying some necessary stress relaxation relations. These equations derived also can be reduced to the case of elementary flexure without shear deformations. Theoretical and simulation results show that the static deformation and dynamic properties are all sensitive to the thermoelasticity and thermo-electricity of the piezoelectric medium. Both the temperature-stress and pyroelectric coefficients decreases the resonance frequency and displacement amplitude evidently. The outcome is widely applicable, and can be utilized to provide theoretical and practical guidance for the design and application of piezoelectric devices especially when considering the temperature response.

1. Introduction

Piezoelectric materials have been made into several kinds of electronic devices, including interdigital transducers, energy harvesters, transformers, gyroscopes, rotary actuators, BAW (bulk acoustic wave) and SAW (surface acoustic wave) sensors (Benes et al., 1998; Stanton et al., 2011; Vellekoop, 1998) and so forth, just because of their inherent electromechanical characteristics, the direct and converse piezoelectric effects. It is well known that the temperature variation can significantly affect the properties of these piezoelectric electronic devices, which owes to the fact that temperature variation induces changes in the material properties (Gubinyi et al., 2008; Yamada et al., 2001). For instance, the elastic properties, piezoelectric coefficients, and dielectric permittivity will change if the piezoelectric materials work at high temperatures (Gubinyi et al., 2008). In extreme cases, piezoelectric properties will gradually fade away if the material temperature approaches the Curie point (Yamada et al., 2001). Therefore, the attempt and exploration about temperature stability of these piezoelectric devices have been extensively carried out during the past decades of years (Ro et al., 2013; Rocas et al., 2013; Tomar et al., 2001, 2005; Tzou and Ye, 1994), which could be better suited for engineering applications.

On the other hand, the thermo-electro-mechanical coupling problem also can be solved in the linear theory of generalized thermoelasticity for piezoelectric materials (Ashida et al., 1997; Chandrasekharaiah, 1988; Mindlin, 1974; Tzou and Ye, 1994), which has ignored the change of material properties caused by small temperature perturbation. A system of two-dimensional equations were firstly derived by Mindlin for high frequency motions of crystal plates accounting for coupling of mechanical, electrical and thermal fields (Mindlin, 1974). Chandrasekharaiah (1988) and Singh (2005) developed constitutive formulations for dynamic linear piezothermoelasticity. Based on these theoretical equations, many issues have been addressed, including nonlinear dynamic behaviors (Fung et al., 2001), wave propagation (Cao et al., 2011; Sharma et al., 2004), crack effect (Ueda, 2006), transient response (Choi et al., 1995), and so on. During the theoretical analysis of the works mentioned above, some model simplifications, more or less, have been adopted. For instance, it is very common to assume that the piezoelectric device is unit size or infinite in one direction in order to make the theoretical derivation and numerical calculation feasible (Cao et al., 2011; Choi et al., 1995; Mindlin, 1974; Sharma et al., 2004). However, the size of a real piezoelectric device is limited. Sometimes, maybe it will lead to incorrect results when using of the infinite size assumption (Son and Kang, 2011).

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For three dimensional solids, finite element method (FEM) can be used to solve the multi-physics coupling problem (Gornandt and Gabbert, 2002). However, owing to material anisotropy and its finite size in three directions for a piezoelectric material, extensional, flexural, shear and torsional deformations usually coupled together. It is hard to completely distinguish these coupling modes from the results obtained by FEM. Hence, based on the classical three-dimensional equations of the linear theory of piezoelectricity, we derived one-dimensional equations of a thermal piezoelectric beam by using of double power series expansion along the thickness and width directions. With the introduction of some necessary stress relaxation relations, the extension and flexure as well as shear constitutive relations are revised. The equations derived can be deduced to some classical outcomes, such as, the case of piezoelectric beam if letting both the temperature-stress and pyroelectric coefficients to be zero, and the case of elementary flexure without shear deformations if setting the zero-order flexural shear strains to be zero. Our theoretical analysis is based on power series expansion, which has been well established by Mindlin (1972, 1974). Dokmeci (1974) and Yang (1998) have separately derived one-dimensional equations by expanding the displacement components and electrical potential function into double series along thickness and width direction for a pure piezoelectric beam. Later, Zhang et al. (2009) furthermore investigate the vibration properties of a piezoelectromagnetic beam using this method. In present contribution, we continue to derive the one-dimensional equation of a piezoelectric beam considering its intrinsic thermal effect and pay more attention to the influence of temperature-stress and pyroelectric coefficients on the static deformation and dynamic properties for single extensional motion. The outcome is expected to provide theoretical and practical guidance on the understanding of thermoelasticity and thermoelectricity in piezoelectric materials, as well as the design and application of piezoelectric devices in temperature fields.

2. Basic three-dimensional equations

Consider a thermo-piezoelectric crystal beam as shown in Fig. 1, whose width, thickness and length are $2a$, $2b$ and $2c$, respectively. The basic mechanical behavior of thermo-piezoelectric crystals can be described by the classical linear theory of thermopiezoelectricity (Ashida et al., 1997; Chandrasekharaiah, 1988; Mindlin, 1974; Tzou and Ye, 1994). The corresponding displacement component, electrical potential function and temperature rise from the stress free reference temperature Θ_0 are denoted by u_i, φ, and θ, respectively. In absence of body force, the linear theory for small and dynamic signals in a thermal piezoelectric material consists of the equations of motion (Newton's law), Gauss's law of electrostatics, and the entropy density equation (Ashida et al., 1997; Chandrasekharaiah, 1988; Mindlin, 1974; Singh, 2005; Tzou and Ye, 1994), i.e.,

$$T_{ij,j} = \rho \ddot{u}_i, \tag{1a}$$

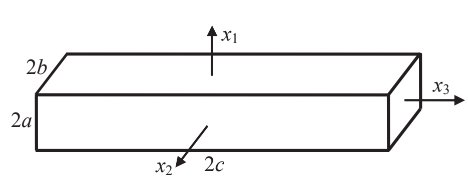

Fig. 1. A thermo-piezoelectric beam with a rectangular cross section and coordinate system.

$$D_{i,i} = 0, \tag{1b}$$

$$-\Theta_0 \dot{\eta} = h_{i,i}, \tag{1c}$$

where T_{ij}, D_i, and η stand for components of stress, electric displacement and entropy density, respectively. ρ is the mass density with h_i being the heat flux. The summation convention for repeated tensor indices is used. Meanwhile, a superimposed dot represents differentiation with respect to time t. Here, we only consider the case of small temperature change, i.e., $\theta << \Theta_0$, and ignore the change of material properties caused by small temperature perturbation. The governing equations above are accompanied by the following constitutive relations (Zhang et al., 2009):

$$S_{ij} = s_{ijkl}T_{kl} + e_{kij}E_k + \lambda_{ij}\theta, \tag{2a}$$

$$D_i = e_{ikl}T_{kl} + \varepsilon_{ik}E_k + p_i\theta, \tag{2b}$$

$$\eta = \lambda_{kl}T_{kl} + p_kE_k + \alpha\theta, \tag{2c}$$

where s_{ijkl}, e_{kij}, and ε_{ij} are the elastic compliances, piezoelectric constant, and dielectric permittivity, with λ_{ij} and p_i being the temperature-stress coefficient and pyroelectric constant, respectively. $\alpha = \rho C_v^E/\Theta_0$, in which C_v^E stands for specific heat capacity. Here, the strain tensor S_{ij}, electric field E_i, and the heat flux h_i are defined by

$$S_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),\ E_i = -\varphi_{,i},\ h_i = \kappa_{ij}\theta_{,j}, \tag{3}$$

in which κ_{ij} stands for heat conduction coefficient. On the boundary of a finite body with a unit outward normal vector $\boldsymbol{n}$, usually the mechanical displacement component u_i or traction vector $T_{ij}n_i$, electric potential function φ or normal component of the electric displacement vector D_in_i, the temperature (θ+Θ_0) or the entropy density may be prescribed to obtain closed-form solutions of the problem.

3. Double power series expansion

It is assumed that the thermo-piezoelectric beam has a slender shape, i.e., $c >> a,b$, with its rectangular cross section as shown in Fig. 1. In order to develop a one-dimensional theory for thermo-piezoelectric beams, we will make the following double power series expansions of the mechanical displacement component u_i, electrical potential function φ and perturbation of temperature θ (Dokmeci, 1974; Mindlin, 1972, 1974; Vashishth and Gupta, 2009; Yang, 1998; Yang et al., 1999; Zhang et al., 2009):

$$\begin{aligned} &u_i = \sum_{m,n=0}^{\infty} x_1^m x_2^n u_i^{(m,n)}(x_3,t),\ \varphi = \sum_{m,n=0}^{\infty} x_1^m x_2^n \varphi^{(m,n)}(x_3,t), \\ &\theta = \sum_{m,n=0}^{\infty} x_1^m x_2^n \theta^{(m,n)}(x_3,t). \end{aligned} \tag{4}$$

These expansions in (4) and the general derivation below include the usual zero-order and first-order theories for extension and flexure in which the beam cross section moves and rotates but does not deform, as well as higher-order effects describing various deformation modes of the cross section (Dokmeci, 1974; Yang, 1998; Yang et al., 1999). Substitution of Equation (4) into Equation (3) yields the corresponding series for the strain tensor S_{ij} and electric field E_i as well as heat flux h_i, i.e.,