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Generalized plate model for highly contrasted laminates

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ABSTRACT

This paper deals with highly contrasted stratified laminates. The effective plate behavior is derived from the 3D constitutive law of the materials combined with an asymptotic expansion formulation and an appropriate scaling the stiffness contrast. This method enables to justify the a priori assumptions used in the previous approaches. The different regimes of behavior are clearly specified, according to the mechanical and geometrical parameters of the layers, and to the loading. The method provides a synthetic bi-torsor representation that facilitates the understanding of the coupling between the shear and bending mechanisms. Applications to laminated glass and hard skin sandwich panels are presented.

1. Introduction

This paper deals with the behavior of Highly Contrasted Sandwich (HCS) or stratified plates. HCS-plates are common in aeronautics where panels with hard skin and soft core are widely used. HCS-plates are also encountered in other engineering fields where stiff layers are glued with a soft matter. This is the case of glass laminates made of glass plies (two or more) bounded by thin layers of soft viscoelastic polymer.

Owing to their plane geometry, plates can be described by two dimensional theories provided that the characteristic length of deformation is significantly larger than the plate thickness. Under this assumption, homogeneous plates have been described historically by the classic Kirchhoff model (analogous to Euler beam) that accounts for bending only. This description has been improved by the Reissner–Mindlin model (analogous to Timoshenko beam) that includes a correction accounting for transverse shear deformation in thick plates (Reissner, 1995). These classical models perform well for homogeneous plates however, their direct extension to laminated plates raised difficulties for modeling properly the action of shear forces. Many suggestions were made that lead to more complex models. A review on these approaches can be found in Carrera (2003) and Reissner (1995). Such models are related to generalized continua as detailed in Altenbach et al. (2010). In this perspective, the new concept of bending gradient recently introduced in Lebée and Sab (2012) draws a promising way to determine the shear effect in thick stratified plates.

The classical models of homogeneous plates have been justified theoretically in Ciarlet and Destuynder (1979) using a multi-scale asymptotic method based on the scale ratio ε between the plate thickness h and the characteristic size of evolution of the phenomena in the plane of the plate L, namely

$$\varepsilon = h/L \ll 1$$

The reader may refer to Ciarlet (1997) and Trabucho and Viano (1996) for a comprehensive analysis of the asymptotic approach. The multi-scale homogenization method has been then extensively used to derive the effective description for periodic plates, corrugated plates and stratified plates (Caillerie, 1982, 1984; Lewinsky and Telega, 2000). The elastic properties of the constituents are assumed of the same order of magnitude, and the inner stress distribution, that determines the effective behavior, results from this assumption. In case of highly contrasted constituents, the inner stress distribution notably differs and the effective behavior has to be reconsidered quantitatively and qualitatively.

Indeed, it is well recognized that the flexural response of laminates is highly influenced by the contrast of rigidity between the different layers. Consider for instance a tri-laminate. A central layer of high stiffness gives a (quasi-) perfect connection and the whole layers behave as a monolithic plate governed by a global bending. Conversely, with very deformable central layer each external layer slides (quasi-)perfectly with each other, and is governed by its own bending. At intermediate values, the shear of the central layer has to be taken into account to provide the transition between the monolithic and multi-layer response.

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Besides, the difference between the two limits is large, so that the limits do not provide a reasonable assessment of the actual behavior, hence, a comprehensive relevant modeling requires encapsulating – at least – the monolithic and multi-layer bending mechanisms and the shear effect. This question is particulary important for laminated glass, because the viscoelastic stiffness of the polymer significantly depends on frequency and temperature. Thus, in practice, the actual behavior may reach either the monolithic or the multi-layer limits, depending on the loading and external conditions. This situation departs from the usual or refined description of elastic stratified plates with a limited level of contrast. In fact, a weak contrast reduces the possible kinematics and therefore eliminates some mechanisms occurring with high contrasts.

The question of HCS-plates has been raised by Berdichevsky (2010) for elastic plates having thin hard skin and thicker soft core. A theoretical universal plate model was proposed, introducing a priori the plate kinematics in each layer, and using an energetic asymptotic approach. Despite the generality of this approach, the transfer of the energy formulation into the usual terms of moment and transverse forces of plate theories remains difficult. The "opposite" morphology, i.e. plies with thin interlayer, has been also studied, mainly in the context of glass-polymer. A number of approximated models has been proposed, e.g. (Asik and Tezcan, 2005; Norville et al., 1998) and a good review can be found in Galuppi and Royer-Carfagni (2012b) or Foraboschi (2012). More refined approaches accounting for the elastic or viscoelastic shear behavior of the interlayer were developed by Ivanov (2006), Galuppi and Royer-Carfagni (2012a), and Foraboschi (2012) for static loading. In these works, the descriptions are established by considering the balance equations of the different layers in which the inner kinematics are *assumed*. By construction, the usual kinematics variables as well as moments and transverse forces of the plate are complemented by additional variables.

More generally, the developments of the last decades on laminated plate theories, provide refined "zig-zag theories" for plates. The main idea of these axiomatic theories is to postulate the adequate "zig-zag" kinematics, enabling to respect as close as possible the continuity and equilibrium equations in each layer. This allows a better account for the shear effect. The comparative study (Carrera and Ciuffreda, 2005), underlines that the "best" choice among the different proposals available in the literature may vary according to the desired accuracy (and possibly the specificity of the plate, the nature of the loading and type of the boundary conditions). The interest, but also the drawback, of these refined models is that the geometry and mechanical properties of the layers are somehow "hidden" in the "zig-zag" kinematics. Thereby they provide accurate numerical results for elastic plates but do not lead to synthetic formulations that enables to handle the different involved mechanisms easily. For instance, in the case of a viscoelastic interlayer, the shear modulus varies over several decades and would require a continuous actualization of the zig-zag kinematics in accordance with the current value of the modulus.

The purpose of this paper is to derive a synthetic quasi-analytic formulation describing the HCS plates under static and dynamic loadings through a multi-scale asymptotic approach combined with a scaling of the parameters. According to the multi-scale asymptotic method (Sanchez-Palencia, 1980; Ciarlet, 1997; Auriault et al., 2009) the effective 2D behavior is derived from the 3D constitutive law of the materials combined with (i) the specific asymptotic expansions in ε power induced by the plate geometry, and (ii) the appropriate scaling of the stiffness contrast. This method allows justifying the a priori assumptions used in previous approaches, and therefore the different regimes of behavior are clearly specified, according to the mechanical and geometrical parameters of the layers and the type of loading (as well as the temperature in the viscoelastic case). The method leads to a bi-torsor representation that facilitates the understanding of the coupling between the shear and bending mechanisms. This is achieved by re-expressing the differential sliding motion between the stiff layers in the form of an overall rotation, and then, by associating this additional kinematic descriptor to an overall moment. In this way the out-of-plane behavior of HCS plates reduces to the following simple form (f stands for the external transverse forces):

$$\begin{cases} \mathrm{div}_x(\mathcal{T}) & = & f \\ \underline{\mathcal{T}} & = & \underline{T} - \underline{\mathrm{div}}_x(\underline{\underline{M}}) \\ \underline{T} & = & -\underline{\mathrm{div}}_x(\underline{\underline{\mathcal{M}}}) \end{cases}$$

where the constitutive laws relating T, $\underline{\underline{M}}$ and $\underline{\underline{\mathcal{M}}}$ to the kinematic variables, namely, the deflection, the deflection's gradient, and the overall rotation, are explicitly determined from the properties and geometry of the three layers.

Results of the present paper are limited to sandwich or symmetric stratified plates with respect to their middle plane. The focus is the response to transverse loads and therefore to the bending/shearing problem. However the in-plane behavior is also derived. Furthermore, the developments are performed in the framework of small deformation and of linear elastic and/or viscoelastic behavior.

The paper is organised as follows. Section 2 is devoted to the scaled formulation of HCS-plates. In Section 3 the HCS-plate model is established through the asymptotic expansions method. The results are discussed in Section 4. Applications to laminated glass and sandwich panels are discussed in Section 5.

2. Scaled formulation of HCS-plates

This section aims to express mathematically the influence of the plate geometry on the formulation, and to scale the physics in the case of high mechanical contrast. We consider a symmetric stratified plate Ω (see Fig. 1) mades of two identical external stiff layers Ω_+ and Ω_-, each of thickness h, perfectly connected to a soft central layer Ω_c of thickness c of the same order of magnitude as h. Thus, $\Omega_t = \Omega_+ \cup \Omega_c \cup \Omega_-$ denotes the whole plate, and $\Omega = \Omega_+ \cup \Omega_-$ denotes the two external layers; $\Gamma = \Gamma_+ \cup \Gamma_-$ represents the external boundaries of Ω while $\Gamma_c = \Gamma_{c_+} \cup \Gamma_{c_-}$ are the boundaries of Ω_c. The characteristic size of evolution of the phenomena in the plane of the plate L is significantly larger than the total thickness of the plate $h_t = 2h + c$.

Herein, the orientation is specified using the reference orthonormal frame $\{\underline{a}, \underline{a}_\alpha\}$, $\alpha = 1, 2$ (by convention, Greek indices run from 1 to 2) and the associated Cartesian coordinates x, x_α, where $\underline{a}$ denotes the out-of-plane direction of the plate and $\underline{a}_\alpha$, $\alpha = 1, 2$ designate the in-plane directions (hence, the in-plane position vector is given by $\underline{x} = x_\alpha \underline{a}_\alpha$). The displacement vector $\underline{\mathrm{u}}$ is decomposed into the deflection, i.e. the out-of-plane component denoted w, and the in-plane components denoted u_α, hence $\underline{\mathrm{u}}(\mathrm{x}, \underline{x}) = w(\mathrm{x}, \underline{x})\underline{a} + \mathrm{u}_\alpha(\mathrm{x}, \underline{x})\underline{a}_\alpha$.

At first, let us focus on the situation where (i) the plate is made of elastic materials (linear isotropic), (ii) the current plate section is assumed free of surface and volume forces, (iii) the plate behaves in quasi-static regime. Under these assumptions, the local governing equations consist in the momentum balance without body force, the isotropic elastic constitutive law, the perfect contact condition on the internal interface and the no-loading condition on the external surfaces. These equations read: