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Cracks at elliptical holes: Stress intensity factor and Finite Fracture Mechanics solution

P. Weißgraeber*, J. Felger, D. Geipel, W. Becker

Technische Universität Darmstadt, Fachgebiet Strukturmechanik, Franziska-Braun-Straße 7, 64287 Darmstadt, Germany

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ABSTRACT

In this work crack initiation at elliptical holes in plates under uniaxial tension is studied by means of a closed form analytical Finite Fracture Mechanics approach. To allow for a detailed study of crack initiation, stress intensity factors available in literature are discussed and compared to extensive numerical results. Based on physical rationale, an improved stress intensity factor solution is proposed that shows a very good agreement with numerical results for a wide range of stress concentration factors of the ellipse and crack lengths. Using the exact solution of the stress field in the notched plate and the proposed stress intensity factor, an efficient Finite Fracture Mechanics solution is obtained. No empiric length parameters are introduced but only the strength and the fracture toughness are required for evaluation. The analysis comprises the case of a circular hole and the limiting cases of a crack tangential and normal to the loading direction. Typical notch size effects are covered by this coupled stress and energy approach. A continuous transition from strength of materials to Linear Elastic Fracture Mechanics can be rendered.

1. Introduction

There are almost no structures without stress concentrators as flaws, notches or holes. Typically static or fatigue failure occurs due to cracks or damage originating from such stress concentrations. An important set of stress concentrators are blunted notches or open holes, as they are often necessary in engineering design of structures. The assessment of crack or damage initiation at such concentrators has been addressed by many researchers. Stress based approaches that evaluate the stresses at specific distances (Whitney and Nuismer, 1974) or fracture mechanics based approaches that assume the existence of an inherent flaw (Waddoups et al., 1971) were proposed. These models depend on a previously defined length parameter, which is not a material parameter but depends on the structural situation and the loading. Using Finite Fracture Mechanics (FFM), this deficiency can be overcome. Then, with evaluating a coupled stress and energy criterion, crack initiation can be assessed and the corresponding crack initiation load and the size of the initiated crack can be identified. FFM has been applied successfully to study failure of many different structural situations, such as the delamination in laminates due to free-edge effects (Martin et al., 2012), analysis of bolted joints (Catalanotti and Camanho, 2013), adhesive joints (Weißgraeber and Becker, 2013; Hell et al., 2014), thermal crack patterns (Leguillon, 2013) and crack initiation at V-notches (Carpinteri et al., 2008). The initiation of short notch-emanated cracks in U-notched structures has also been studied by Carpinteri et al. (2012).

To assess stress concentrators in the framework of FFM, the corresponding stress intensity factors of cracks emanating from these points must be available. Only for few certain structural situations exact solutions can be given. In most cases only approximate formulae for small cracks are provided that are based on numerical solutions such as Boundary Collocation Method, Mapping Function Method, Boundary Element Method or Finite Element Method. Despite modern computers and efficiency of modern methods as the universal Finite Element Method, such approximate solutions are still very important to efficiently analyse failure of structures and to study potential parameter dependencies.

In the present work the elementary case of an elliptical hole with semi-axes a and b under uniaxial loading shall be studied in detail. The analysis comprises the important case of a circular hole as well as the limiting cases of the ellipse forming a crack tangential to the loading direction without stress concentration or a crack normal to the loading direction with singular stresses.

* Corresponding author.
E-mail address: weissgraeber@fsm.tu-darmstadt.de (P. Weißgraeber).

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2. Elliptical hole in a plate with symmetrical cracks

In this work an elliptical hole with semi-axes a and b in a plate under uniaxial tension σ_∞ shall be considered, cf. Fig. 1. The stress distribution can be obtained with the complex potentials method (Inglis, 1913; Timoshenko and Goodier, 1951). Using a transformation from elliptical coordinates to Cartesian coordinates the normal stresses in loading direction σ_{yy} ahead of the ellipse ($x \geq a$, $y = 0$) can be written as:

$$\hat{\sigma}_{yy} = \frac{\sigma_{yy}}{\sigma_\infty} = \frac{\left(\frac{a}{b}+1\right) + \frac{a}{b}\frac{x}{\sqrt{x^2-a^2+b^2}}\left(\left(\frac{a}{b}\right)^2 - \frac{a}{b} - 3 + \frac{x^2}{x^2-a^2+b^2}\right)}{\left(\frac{a}{b}+1\right)\left(\frac{a}{b}-1\right)^2}. \tag{1}$$

Evaluating this solution at the boundary of the ellipse ($x = a$) leads to the well-known stress concentration factor $k_t = 1 + 2a/b$ of an elliptical hole. However, the solution (1) is singular for circular holes ($a/b = 1$). In this case the classical solution by Kirsch (1898) for circular holes can be used:

$$\hat{\sigma}_{yy} = \frac{\sigma_{yy}}{\sigma_\infty} = \frac{1}{2}\left(2 + \left(\frac{a}{x}\right)^2 + 3\left(\frac{a}{x}\right)^4\right). \tag{2}$$

2.1. Stress intensity factor of symmetric cracks emanating from the elliptical hole

Considering the stress intensity factor of cracks that emanate normal to the loading direction, so far no closed-form solution is available.

Several researchers, however, have provided numerical results. Bowie (1956) has obtained results using polynomial approximations of mapping functions for the method of complex potentials for the case of a circular hole. Newman (1971) (Boundary Collocation Method) and Nisitani and Isida (1973) (Body Force Method) studied the case of elliptical holes and provided a total of 89 values for several configurations in the range: $1/4 \leq a/b \leq 4$ and $0.02 \leq s = \Delta a/(\Delta a + a) \leq 0.58$. Based on these results for elliptical notches Tada et al. (1985) have provided the stress intensity factor as a design chart. The same results were used by other researchers to develop approximate formulae of the type $K_I = F\sqrt{\pi\Delta a}$ (Schijve, 1980; Lukáš, 1987; Kujawski, 1991; Kotousov and Jones, 2002). The solution by Lukáš (1987)

$$\hat{K}_I = \frac{K_I}{\sigma_\infty} = \frac{1.122\left(1+2\frac{a}{b}\right)}{\sqrt{1+4.5\frac{\Delta a\, a}{b^2}}}\sqrt{\pi\Delta a} \tag{3}$$

yields the best approximation and has been used most frequently but the validity is restricted to short notch-emanating cracks. In Lukáš (1987), the approximation of the stress intensity factor is compared with numerical data of Newman (1971) for different crack lengths and stress concentration factors k_t up to $k_t = 9$. For $\Delta a/R \leq 1/3k_t$, the error of formula (3) is less than 5%, where $R = b^2/a$ is the root radius. To the authors' knowledge no detailed comparison for a larger range of configurations is given in literature.

The stress intensity factor (3) correctly contains the case of a small crack emanating from a shallow notch, whose stress intensity factor is

$$\hat{K}_I = 1.122 k_t \sqrt{\pi\Delta a} \tag{4}$$

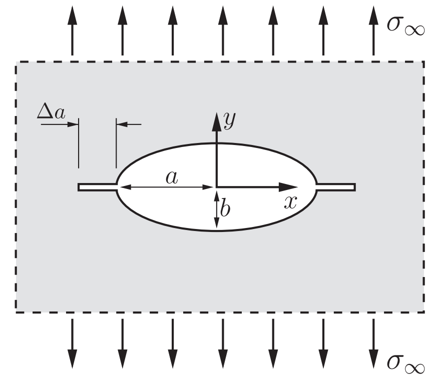

Fig. 1. Ellipse with semi-axes a and b under uniaxial tension σ_∞ with symmetrical cracks of length Δa normal to the loading direction.

but it does not revert to the limiting case of a centre crack for $b \to 0$

$$\hat{K}_I = \sqrt{\pi(a+\Delta a)}, \tag{5}$$

instead

$$\lim_{b\to 0} \hat{K}_I = 1.06\sqrt{\pi\Delta a} \tag{6}$$

is obtained.

To propose an improved stress intensity factor, the formula by Lukáš (1987) was considered and it was aimed to correctly cover the limiting cases of shallow and deep cracks at notches and the limiting case of a centre crack. Hence, a transition of a stress intensity factor solution proportional to the square root of the crack length to a solution proportional to square root of the notch depth plus the crack length must be obtained. Based on these considerations, we propose an improved stress intensity factor that correctly contains the limiting cases of a centre crack $b \to 0$ and an edge crack for $b \to \infty$:

$$\hat{K}_I = \frac{K_I}{\sigma_\infty} = \frac{1.122\left(1+2\frac{a}{b}\right)}{\sqrt{1+5.04\frac{\Delta a\, a}{b^2}\left(\frac{a}{a+\Delta a}\right)^{1-\tanh(3b/(2\Delta a))}}}\sqrt{\pi\Delta a}. \tag{7}$$

The replacement of the factor 4.5 in the original formula by Lukáš to improve the fitting for short cracks has also been done by Carpinteri et al. (2011). An extension of Lukáš's formula for rounded V-notches can be found in Sapora et al. (2014).

To validate the stress intensity factor and identify the range of validity, Finite Element Analyses with Abaqus 6.13 are performed. A quarter model is employed as shown in Fig. 2. A very fine meshing is used with a fine radial mesh at the crack tip. Convergence is checked: The number of elements and the outer dimensions L_1, L_2 are chosen such that they do not affect the results (effect smaller than 0.1%). Continuum plane stress elements are used. The elements at the crack tip have a characteristic size of 2×10^{-4} mm and the models have around 2×10^4 degrees of freedom. The stress intensity factor is calculated by evaluating the J-integral (Rice, 1968) on a contour around the crack tip. More than 1300 numerical calculations were performed for a wide range of aspect ratios of the ellipse (a/b) and related crack lengths $s = \Delta a/(a + \Delta a)$.

Fig. 3 shows the numerical results compared to the stress intensity factor (3) by Lukáš (1987) and the proposed improved formula (7). For small aspect ratios and crack lengths both solutions show very good agreement but with raising crack lengths or aspect ratio only the proposed formula yields a good agreement with the