European Journal of Mechanics A/Solids 55 (2016) 199–211

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

# Optimal control of laminated plate integrated with piezoelectric sensor and actuator considering TSDT and meshfree method

R. Talebitooti [a, *], K. Daneshjoo [a], S.A.M. Jafari [b]

[a] Sch. of Mech. Eng., Iran Univ. of Sci. and Tech., Teheran 16844, Iran
[b] Sch. of New Technologies, Iran Univ. of Sci. and Tech., Teheran 16844, Iran

ARTICLE INFO

*Article history:*
Received 21 April 2015
Accepted 10 September 2015
Available online 25 September 2015

*Keywords:*
Optimal control
Meshfree method
TSDT theory

ABSTRACT

In this paper, the element free galerkin (EFG) method based on third-order shear deformation theory (TSDT) is used to investigate shape and vibration control of piezoelectric laminated plate bonded with piezoelectric actuator and sensor layers. The electric potential distributions through the thickness for each piezoelectric layer are assumed to vary linearly. In addition, a closed-loop velocity feedback control and optimal steady-state regulator with output feedback algorithm is used for the active control of the static deflection as well as the dynamic response of the plates with bonded distributed piezoelectric sensors and actuators. Furthermore, the effects of the size of support and nodal density on the numerical accuracy are also investigated. The results indicate that, the accuracy and reliability of presented work have an excellent agreement with those of other available numerical approaches such as finite element and FSDT meshfree method.



## 1. Introduction

The study of embedded or surface-mounted piezoelectric materials in structures has received considerable attention in recent two decades (Mitchell and Reddy, 1995). It is due to possibility of creating certain types of structures and systems capable of adapting to or correcting for changing operating conditions. The advantage of incorporating these special types of material into the structure is due to the fact that the sensing and actuating mechanism becomes part of the structure by sensing and actuating strains directly.

Classical mathematical models have already been well established for the phenomena in the areas of mechanics of solids and structures. Meanwhile, different types of differential or partial differential equations (PDEs) that govern these phenomena have also been derived. There are largely two categories of numerical methods for solving these PDEs (Liu and Gu, 2005): direct approach and indirect approach. The direct approach known as strong form methods (such as the finite difference method (FDM) and collocation method) discretizes and solves the PDEs directly, and the indirect approach known as weak form methods (such as finite element method (FEM)) establishes first an alternative weak form system equation that governs the same physical phenomena and then solves it. The weak form equations are usually in an integral form, implying that they need to be satisfied only in an integral (averaged) sense. The (EFG), one of known category of meshfree, is a standard weak formulation that is variationally consistent due to the use of compatible moving least squares (MLS) shape functions and the Galerkin approach with constraints to impose the essential boundary conditions. In the EFG method, in order to derive the stiffness matrix, some complex integrals should be solved via numerical procedures. Therefore, a need for background mesh to perform the numerical integration is unavoidable. Subsequently, in the EFG method the integration cells do not require to be attuned with the scattered nodes distributed on the problem domain. Hence, the generation of background mesh is more easily than the FEM methods. Liew et al. (2004, 2002) represented a formulation by employing the EFG method based on the first-order shear deformation theory to study the shape control and vibration suppression of piezo-laminated composite beams and plates. Liu et al. (2004) developed the previous work with employing the point interpolation method using radial basis functions (RPIM) based on the first-order shear deformation theory.

In EFG method, the shape functions constructed by MLS approximation do not consider the property of delta functions. Hence, the essential boundary conditions cannot be imposed as conveniently as the standard FEM method. Liu and Chen (2001)

* Corresponding author.
*E-mail address:* rtalebi@iust.ac.ir (R. Talebitooti).

http://dx.doi.org/10.1016/j.euromechsol.2015.09.004
0997-7538/

consider the orthogonal transformation techniques to impose the essential boundary conditions for free vibration analysis of a thin plate. Dai et al. (2004) used the penalty method to impose the essential boundary conditions for the static deflection and free vibration analysis of composite plates. This proposed method presented by Dai et al. (2004) is more efficiently than orthogonal transform method (Liu and Chen, 2001).

Samanta et al. (1996) represented a generalized finite element formulation with an eight noded two-dimensional quadratic quadrilateral isoparametric element for active vibration control of a laminated plate integrated with piezoelectric polymer layers acting as distributed sensors and actuators based on third-order shear deformation theory. Similar works developed finite element formulation for active vibration control of smart structure (Moita et al., 2005; Phung-Van et al., Sep. 2013; Phung-Van et al., 2014; Thinh and Ngoc, 2010).

The modern control techniques are to be widespread in designing the stability augmentation systems. This is accomplished by regulating certain states of the system to zero while obtaining desirable closed-loop response characteristics. Output feedback will allow engineers to design plant controllers of any desired structure. In the constant gain velocity feedback control strategy (CGVF), the stability of the system is guaranteed only when the actuators and sensors are truly collocated. Moreover, the linear quadratic regulator (LQR) algorithm does not require collocated actuator–sensor pairs for stability, but it requires the measurement of all state variables, which is a difficult proposition. This is another reason for preferring output feedback over full-state feedback (Lewis et al., 2012). Ray (1998) presented a simple method of closed-form solution for optimal control of thin symmetric laminated plates with output feedback using distributed piezoelectric sensors and actuators. Bhattacharya et al. (2002) proposed a new control strategy based on Independent Modal Space Control (IMSC) technique and used for the vibration suppression of spherical shells made of laminated composites. Zabihollah et al. (2007) investigated the vibration control of the new generation of smart structures using the LQR strategy with finite element model based on the layerwise theory of Reddy (2004). Kusculuoglu and Royston (2005), Kapuria and Yasin (2010) used a reduced-order state space model for active vibration suppression of piezoelectric laminate plates using both classical (CGVF) and optimal control strategies (LQR).

In the literature reviewed above in active vibration control of piezoelectric composite plates with meshfree method, the first order shear deformation theory are used, whereas this theory for thick plate and high frequency response is less accurate. Thus, in present job the higher order theory is accomplished. Controllers used, in above literature are not appropriate. In some works, the CVGF controller is employed which is a simple controller, but the stability of the system is not guaranteed in general conditions. Moreover, in some works the LQR controllers are used. In spite of efficient stability LQR controller, it requires the measurement of all state variables, which is essentially troublesome. In the present work, in order to cope with this recent drawback, the LQR with output feedback is recommended. In this controller both stability of the system as well as the swiftness of the time response expenditure are satisfied. Therefor the objective of this paper is to develop the EFG method based on the third-order shear deformation theory for static, free vibration and dynamic control of piezoelectric composite plates integrated with sensors and actuators. The active vibration control capability is studied using a simple, CGVF and LQR with output feedback. The accuracy and reliability of the proposed method is verified by comparing its numerical predictions with those of other available numerical approaches.

## 2. Theory and formulation

### 2.1. Linear piezoelectric constitutive equations

The quasi-static linear piezoelectric constitutive equations can be defined as:

$$\{D\} = [e]\{\varepsilon\} + [k]\{E\} \tag{1}$$

$$\{\sigma\} = [c]\{\varepsilon\} - [e]^T\{E\}. \tag{2}$$

where {D}, {E}, {σ} and {ε} represent the vectors of electric displacement, electric field, stress, and strain, respectively; and [c], [k], [e] denote the matrices of plane-stress reduced elastic for a constant electric field, dielectric constant at constant mechanical strain and piezoelectric stress constant, respectively.

The electric field potential relation is given by:

$$E_{x_i} = -\frac{\partial \phi}{\partial x_i} \tag{3}$$

The plane-stress elastic constants [c] are given as:

$$[c] = \begin{bmatrix} c_{11} & c_{12} & 0 & 0 & 0 \\ c_{21} & c_{22} & 0 & 0 & 0 \\ 0 & 0 & c_{66} & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & c_{55} \end{bmatrix} \tag{4}$$

where the material constants are given by:

$$c_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, \quad c_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}, \quad c_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}},$$
$$c_{66} = G_{12}, \quad c_{44} = G_{13}, \quad c_{55} = G_{23}\sqrt{a^2+b^2} \tag{5}$$

in which $E$ is Young moduli, $G$ is the shear moduli and ν is the Poisson's ratios.

### 2.2. Displacements and strains based on TSDT

For the TSDT, we assume the following displacement field (Reddy, 2004):

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\varphi_x(x,y) - z^3 c_1\left(\varphi_x + \frac{\partial w_0}{\partial x}\right) \\ v(x,y,z) &= v_0(x,y) + z\varphi_y(x,y) - z^3 c_1\left(\varphi_y + \frac{\partial w_0}{\partial y}\right) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{6}$$

where $c_1 = 4/3\ h^2$ and $(u_0,v_0,w_0)$ denote the displacements of a point on the midplane in (x,y,z) direction and $(\varphi_x,\varphi_y)$ are rotation about (y,x) respectively.

The in-plane strains are thus expressed by the following equation:

$$\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{yy} & \gamma_{xy} \end{bmatrix}^T = \varepsilon^{(0)} + z\varepsilon^{(1)} + z^3\varepsilon^{(3)} \tag{7}$$

where: