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A nonlinear analysis of thermal stresses in an incompressible functionally graded hollow cylinder with temperature-dependent material properties

Amin Moosaie*

Department of Mechanical Engineering, Yasouj University, 75914-353 Yasouj, Iran

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ABSTRACT

A nonlinear thermoelastic analysis of a thick-walled cylinder made of functionally graded material is performed. The dependence of material properties on temperature is taken into account. This makes the governing equations nonlinear. Thus, the perturbation technique is employed to solve the nonlinear heat conduction equation analytically. The so-obtained temperature field is then supplied to elasticity equations which are solved exactly for the case of incompressible elastic material to get displacement and stress distributions. Finally, the temperature field, material properties and radial stress versus the radial direction are plotted and discussed.

1. Introduction

Functionally graded materials (FGM) are a class of advanced composites whose physical (mechanical, thermal, etc.) properties vary smoothly and continuously in space. That is the material properties are continuous functions of spatial coordinates. These materials are useful in applications when different requirements are to be met by the material. If these requirements are diverse, which can arise in many engineering applications, e.g. in aerospace and biomedical applications, then it might occur that one single material fails to satisfy all requirements. Fiber-reinforced and laminated composites are one solution for this ongoing and ever-increasing demand. Despite their usefulness in many areas, they are prone to stress concentration due to material discontinuities as well as damages like delamination. For example, in aerospace applications, more specifically during atmospheric reentry of a space vehicle, the vehicle structure is subjected to mechanical as well as thermal loads. Thus, the material needs to withstands both kinds of loads. Metals like steel and aluminum possess good strength under mechanical loads. However, when it comes to thermal loads at very high temperatures, they are prone to creep and even melting. On the other hand, ceramics are generally good in resisting thermal loadings, but they are often fragile under mechanical loads. So, the structure can be made of a synthetic material which is a blending of both steel and ceramics. That is, the outer surface subjected to aerodynamic heating is made of ceramics and when moving through the thickness towards the inner layers of the structure, the material gradually and continuously changes to steel which is very good at withstanding mechanical loads. This is only one example, but such demands constantly arise in advanced technologies nowadays.

Since FGM's are often to be used under thermal loadings, the thermoelastic analysis of structures made of FGM's is of paramount importance. Many authors have addressed such a thermal analysis and their works can be browsed in the literature. Using perturbation technique, Obato and Noda (1994) analyzed steady one-dimensional thermal stresses in hollow cylindrical and spherical objects. Ootao et al. (1995) made a transient analysis of thermal stresses in a functionally graded hollow cylinder due to a moving heat source in the axial direction. Lutz and Zimmerman (1996), Zimmerman and Lutz (1999) gave the analytical solution for the thermal stresses in thick cylindrical and spherical shells made of FGM's graded in the radial direction. Tanigawa et al. (1999) analytically investigated thermal stresses in a FG semi-infinite body graded with a power function in the thickness direction. Jabbari et al. (2002, 2003) considered an FG hollow cylinder graded by a power function in the radial direction and developed analytical solutions for one- and two-dimensional thermal stresses. Liew et al.

* Tel.: +98 743 322 1711.
E-mail address: moosaie@yu.ac.ir.

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(2003) presented an analysis of thermoelastic problem in an FG hollow cylinder. They proposed a novel technique by which they derived the solution for an inhomogeneous material from the solution for a homogeneous material. You et al. (2005) gave an analytical solution for elastic stresses in thick-walled spherical vessels under internal pressure. They considered the shell to be made of a functionally graded material confined between two inner and outer homogeneous layers. Chen and Lin (2008) performed an elastic analysis of a thick-walled FGM cylindrical shell graded exponentially in the radial direction. Peng and Li (2010) proposed a method to solve steady thermal stresses in a hollow cylinder made of functionally graded materials with physical properties varying in the radial direction.

The aforementioned works are just few examples from the large body of literature addressing mechanical and thermal stresses in thick-walled cylindrical and spherical shells. However, all these works address functionally graded materials with temperature-independent material properties. However, the physical reality for both homogeneous and functionally graded materials is the dependence of material properties on temperature. This is more pronounced especially when the large temperature differences are involved in the problem. That is, the dependence of physical properties on temperature becomes negligible at lower temperature differences. Nevertheless, there are certain applications in which one cannot neglect this fact and has to take temperature-dependent material properties into account. But, this makes the analysis very complicated as the governing differential equations become nonlinear. On the author's knowledge, so far only numerical solutions are used to perform such a full nonlinear analysis. Awaji and Sivakumar (2001) studied the transient thermal stresses of a FGM hollow circular cylinder cooled by surrounding medium using the finite difference method. Azadi and Azadi (2009) presented a transfinite element method for transient analysis of thermal stresses in a functionally graded hollow cylinder with temperature-dependent material properties.

Analytical solution of thermal stresses with temperature-dependent material properties has been always an interesting research topic and a number of works have been published on this matter addressing homogeneous materials, e.g. (Trostel, 1958). In this paper, an analytical solution of the nonlinear thermal and thermoelastic problem for an FGM thick-walled cylindrical shell with temperature-dependent material properties is presented. For this purpose, the temperature field is obtained using perturbation technique. Then, the thermoelastic problem is solved exactly for the case of $\nu = 0.5$ with ν being the Poisson ratio. On the author's knowledge, it is for the first time that such a nonlinear analysis for FGM shells is presented using analytical tools.

The remainder of this paper is organized as follows. Section 2 presents the nonlinear governing equations along with the required boundary conditions. The analytical solution of heat conduction and thermoelastic problems are discussed in Section 3. Section 4 contains the results and discussions and finally the paper is concluded in Section 5.

2. Governing equations and boundary conditions

In this section, the governing differential equations of temperature field and thermal stresses in a hollow cylinder made of a functionally graded material with temperature-dependent properties are presented and discussed. First we shall introduce the FG model which is used to describe the spatial variation of material properties within the cylinder.

For a cylinder which is functionally graded in the radial direction r, the temperature-dependent material properties can be written as $E = E(\vartheta, r)$, $\alpha_t = \alpha_t(\vartheta, r)$ and $\lambda = \lambda(\vartheta, r)$ with ϑ and r being the temperature field and the radial direction, respectively. In this paper, we propose a separable model in which $E = g_1(\vartheta)f_1(r)$, $\alpha_t = g_2(\vartheta)f_2(r)$ and $\lambda = g_3(\vartheta)f_3(r)$. In the literature for homogeneous materials such as steel, the functions g_1 and g_2 are taken to be linear functions of temperature whereas g_3 is considered both linear (Nowinski, 1959) and quadratic (Stanisic and McKinley, 1962) functions of ϑ.

$$E = E(\vartheta, r) = \left(E_0 - E_1\vartheta - E_2\vartheta^2\right)f_1(r), \tag{1}$$

$$\alpha_t = \alpha_t(\vartheta, r) = (\alpha_{t0} + \alpha_{t1}\vartheta)f_2(r), \tag{2}$$

$$\lambda = \lambda(\vartheta, r) = (\lambda_0 - \lambda_1\vartheta)f_3(r), \tag{3}$$

in which E, α_t and λ are respectively the elasticity modulus, thermal expansion coefficient and heat conductivity, and $f_i(r)$ is the grading function. E_0, E_1, E_2, α_{t0}, α_{t1}, λ_0 and λ_1 are material constants. The Poisson ratio ν is assumed to be constant for FG materials. In the present study, we consider an incompressible elastic material with $\nu = 0.5$. This assumption is necessary to obtain an exact solution of thermoelasticity equations. In a future investigation, we will address thermal stresses in an FG material with an arbitrary Poisson ratio by using a perturbation solution of thermoelasticity equations.

In this work, we consider an FGM with a grading function of the form $(r/R_o)^{m_i}$. Thus, the above equations can be rewritten as

$$E = E(\vartheta, r) = \left(E_0 - E_1\vartheta - E_2\vartheta^2\right)(r/R_o)^{m_1}, \tag{4}$$

$$\alpha_t = \alpha_t(\vartheta, r) = (\alpha_{t0} + \alpha_{t1}\vartheta)(r/R_o)^{m_2}, \tag{5}$$

$$\lambda = \lambda(\vartheta, r) = (\lambda_0 - \lambda_1\vartheta)(r/R_o)^{m_3}. \tag{6}$$

2.1. Heat conduction

In this section, the governing equations of steady heat conduction in a functionally graded hollow cylinder with temperature-dependent heat conductivity are presented. These equations can be obtained by considering the energy conservation law, the Fourier constitutive equation for heat conduction and the dependence of heat conductivity on spatial coordinates and temperature as pointed out in Equation (6). Thus, we start with the steady energy conservation law without heat generation:

$$\nabla \cdot \mathbf{q} = 0, \tag{7}$$

where ∇ and $\mathbf{q}$ are the nabla operator and heat flux vector, respectively. The Fourier constitutive law along with the energy conservation equation (7) gives the following field equation for the temperature:

$$\nabla \cdot (\lambda \nabla \vartheta) = 0. \tag{8}$$

This equation for an infinitely long cylinder with axisymmetric temperature field reads

$$\frac{1}{r}\frac{d}{dr}\left(r\lambda(\vartheta, r)\frac{d\vartheta}{dr}\right) = 0. \tag{9}$$

In order to be solved, this equation needs two boundary conditions in r direction. In this paper, we consider prescribed temperatures at inner and outer surfaces of the cylinder: