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A nonlinear homogenization procedure for periodic masonry

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ABSTRACT

The present paper deals with the problem of the determination of the in-plane behavior of masonry material. The masonry is considered as a composite material composed by a regular distribution of blocks connected by horizontal and vertical mortar joints. The overall constitutive relationships of the regular masonry are derived by a rational micromechanical and homogenization procedure. Linear elastic constitutive relationship is considered for the blocks, while a new special nonlinear constitutive law is proposed for the mortar joints. In particular, a mortar constitutive law, which accounts for the coupling of the damage and friction phenomena occurring during the loading history, is proposed; the developed model is based on an original micromechanical analysis of the damage process of the mortar joint. Then, an effective nonlinear homogenization procedure, representing the main novelty of the paper, is proposed; it is based on the transformation field analysis, using the technique of the superposition of the effects and the finite element method. The presented methodology is implemented in a numerical code. Finally, numerical applications are performed in order to assess the performances of the proposed procedure in reproducing the mechanical behavior of masonry material.

1. Introduction

The development of adequate stress analyses for masonry structures represents an important task not only for verifying the stability of masonry constructions, as old buildings, historical town and monumental structures, but also to properly design effective strengthening and repairing interventions.

The analysis of masonry structures is not simple at least for two reasons: the masonry material presents a strongly nonlinear behavior, so that linear elastic analyses generally cannot be considered as adequate; the structural schemes, which can be adopted for the masonry structural analyses, are more complex than the ones adopted for concrete or steel framed structures, as masonry elements are often modeled by two- or three-dimensional elements. As a consequence, the behavior and the analysis of masonry structures still represents one of the most important research field in civil engineering, receiving great attention from the scientific and professional community.

Several numerical techniques have been developed to investigate and to predict the behavior of masonry structures. In fact, in the last decades, the scientific community has demonstrated great interest in the development of sophisticated numerical tools as an opposition to the tradition of rules-of-thumb or empirical formulae adopted to evaluate the safety of masonry buildings. In particular, nonlinear models implemented in suitable finite elements formulations currently represent the most common advanced strategy to simulate the structural behavior of masonry structures.

The main problem in the development of accurate stress analyses for masonry structures is the definition and the use of suitable material constitutive laws. Taking into account the heterogeneity of the masonry material, which results from the composition of blocks connected together by mortar joints, several models have been proposed in literature.

In the framework of the finite element approach in which the mortar joints are modeled by special interface elements and the bricks are characterized by a linear or nonlinear response, among the others, Lofti and Shing (1994) developed a dilatant interface model able to simulate the initiation and propagation of mortar fracture under combined normal and shear stresses in both tension-shear and compression-shear states. Giambanco et al. (2001) proposed an interface model considering a yield surface governed by the classical bilinear Coulomb condition, completed by a tension cut-off; they considered also the dilatancy related to the roughness of contact surfaces resulting after joint fracture. Lourenço and Rots (1997) and Oliveira and Lourenço (2004) developed an interface constitutive model based on the plasticity theory able to reproduce the response of the masonry subjected to cyclic loading histories. Gambarotta and Lagomarsino (1997a) proposed a damage-plastic continuum model for the mortar material and developed a finite element procedure for the simulations of experimented walls subjected to cyclic horizontal actions superimposed on vertical loads. Alfano and Sacco (2006) developed a damage–friction coupled model able to reproduce crack propaga-

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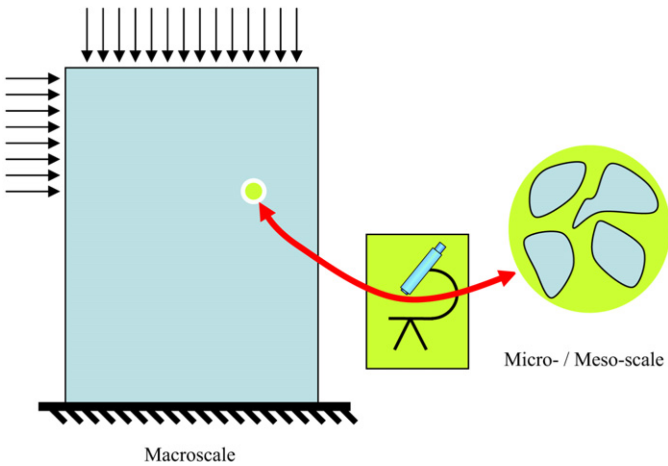

Fig. 1. Scheme of the multiscale analysis for heterogeneous materials.

tion in mode I, mode II, and mixed mode, and to account for the crack initiation and propagation, and for its closure and reopening.

The finite element approach which models separately the bricks and the mortar, even by means of interface concept, leads to structural analyses characterized by great computational effort; this approach can be successfully adopted for reproducing laboratory tests.

The macromechanical models are very effective from a computational point of view when structural analyses are performed; macromechanical models are based on the damage continuum mechanics or on the plasticity theory (e.g. Lourenço, 1998; Berto et al., 2002), even in the framework of nonlocal constitutive laws (Addessi et al., 2001), or they are developed introducing the so-called no-tension material assumption (Del Piero, 1989; Marfia and Sacco, 2005). Indeed, these models are not able to account for the micro-mechanisms occurring in the masonry composite material, or, at least, they consider the failure micro-mechanisms in a very rough way.

The micro-macro models appear very appealing, as they allow to derive in a rational way the stress–strain relationship of the masonry, accounting in a suitable manner for the mechanical properties of each material component and considering the failure micro-mechanisms of the masonry, which play a fundamental role in the overall behavior of the material (Hilsdorf, 1969).

With the reference to Fig. 1, a micro-macro procedure, i.e. a multiscale algorithm implemented into a finite element method framework, could perform the following scheme. Once the actual solution of the structural problem is determined in terms of nodal displacement parameters, the strain can be evaluated at the typical Gauss point of any finite element. The strain computed at structural level is considered as the average strain acting on the Representative Volume Element (RVE) at the microscale level. Then, taking into account the inelastic effects determined during the strain history, the micromechanical problem is solved updating the damage and plasticity state of the RVE and determining the overall behavior of the RVE. It could be emphasized that the determination of the solution of the micromechanical problem governed by inelastic material response of the phases is not a simple task, especially for complex geometrical microstructures, like in the case of masonry material. The micromechanical problem has to be solved at each nonlinear iteration of each Gauss point of each structural finite element, saving all the required history variables. It can be remarked the great computational effort deriving from the development of a multiscale analysis as the one briefly described above. Despite the computational difficulties, the multiscale approach for the study of masonry structures represents a challenging problem and many advances have been done in last ten years. For this reason the development of simple and effective techniques able to solve the nonlinear homogenization problem received great attention.

Different homogenization procedures have been proposed in the literature in order to evaluate the nonlinear behavior of composites. A classic approach is the so-called deformation theory of plasticity, which is based on the use of nonlinear elastic relations approximating the actual incremental constitutive relations of the phases; in this framework, secant and tangent formulations as well as second order estimates have been proposed (Ponte Castañeda, 1996). Moreover, the existence of a potential allows to obtain rigorous bounds by variational methods, such as the generalization of the Hashin–Shtrikman technique. On the other hand, the use of deformation theory of plasticity can lead to unsatisfactory results as it does not account for the dissipation energy and cyclic deformation histories cannot be performed.

An interesting approach is the transformation field analysis (TFA) proposed initially by Dvorak (1992) and adopted and extended to plasticity and viscoplasticity problems by Fish and Shek (2000). According to TFA approach, the inelastic strain, i.e. the transformation strain, is assumed to be uniform within each individual phase in the composite. Chaboche et al. (2001) improved the TFA for deriving the nonlinear behavior of damaging composites, subdividing each phase into subdomains, at the expense of increasing the complexity of the model.

More recently, Lahellec and Suquet (2007) presented a new method for determining the overall behavior of nonlinear composite materials; the method is based on the minimization of an incremental energy function, within an implicit time-discretization scheme, proving that this approach is equivalent to a transformation field analysis with a nonuniform inelastic strain field.

The use of the TFA requires the computation of localization and transformation tensors. To this end, numerical techniques based on generalizations of the Eshelby method (Mura, 1987) can be quite successfully adopted for composites characterized by random microstructure. When the geometry of the microstructure is complex, as in the case of masonry material, numerical techniques, most often based on the assumption of microstructure periodicity, can be adopted. In fact, finite element methods or fast Fourier transform solutions are able to describe accurately local stress and strain fields, so that the correct nonlinear behavior of the phases can be determined.

Simplified micromechanical approaches, derived for the particular microstructural geometry of masonry material, have been developed, among the others, by Kralj et al. (1991), by Pietruszczak and Niu (1992), by Gambarotta and Lagomarsino (1997b) and by Uva and Salerno (2006).

An interesting numerical procedure, based on a multilevel computational approach, has been recently proposed by Brasile et al. (2007). The strategy, which can be reframed in the so-called coupled multiscale analysis (Fish and Shek, 2000), considers two schemes: a local one describing the nonlinear mechanical interaction of the bricks and the mortar joints; a global one, approaching the macromechanical problem via finite element approximation.

With the aim of reproducing the behavior of masonry panels, in this paper a micro-macro modeling approach is presented. In fact, a damage model for masonry material characterized by periodic structures is derived from micromechanical analysis. For old masonries the strength of the mortar is lower than the strength of the bricks. Thus, it can be assumed that damage can develop only in the mortar material (Luciano and Sacco, 1997, 1998a).

In particular, a linear elastic behavior is considered for the blocks and a new special nonlinear constitutive law is developed for the mortar joints. The mortar constitutive law accounts for the coupling of the damage and friction phenomena occurring in the mortar joints during the loading history; the proposed model is