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Internal springs distribution for quasi brittle fracture via Symmetric Boundary Element Method

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ABSTRACT

In this paper the symmetric boundary element formulation is applied to the fracture mechanics problems for quasi brittle materials. The basic aim of the present work is the development and implementation of two discrete cohesive zone models using Symmetric Galerkin multi-zone Boundary Elements Method. The non-linearity at the process zone of the crack will be simulated through a discrete distribution of nodal springs whose generalized (or weighted) stiffnesses are obtainable by the cohesive forces and relative displacements modelling. This goal is reached coherently with the constitutive relation $\boldsymbol{\sigma} - \boldsymbol{\Delta u}$ that describes the interaction between mechanical and kinematical quantities along the process zone.

The cracked body is considered as a solid having a "particular" geometry whose analysis is obtainable through the displacement approach employed in [Panzeca, T., Salerno, M., 2000. Macro-elements in the mixed boundary value problems. Comp. Mech. 26, 437-446; Panzeca, T., Cucco, F., Terravecchia, S., 2002b. Symmetric Boundary Element Method versus Finite Element Method. Comput. Meth. Appl. Mech. Engrg. 191, 3347-3367] by some of the present authors in the ambit of the Symmetric Galerkin Boundary Elements Method (SGBEM). In this approach the crack edge nodes are considered distinct and the analysis is performed by evaluating all the equation system coefficients in closed form [Guiggiani, M., 1991. Direct evaluation of hypersingular integrals in 2D BEM. In: Proceedings of the 7th GAMM Seminar on Numerical Techniques for Boundary Element Methods. Kiel, Germany; Gray, L.J., 1998. Evaluation of singular and hypersingular Galerkin boundary integrals: direct limits and symbolic computation. In: Sladek, J., Sladek V. (Eds.), Singular Integrals in Boundary Element Methods, Computational Mechanics Publications, Southampton; Panzeca, T., Fujita Yashima, H., Salerno, M., 2001. Direct stiffness matrices of BEs in the Galerkin BEM formulation. Eur. J. Mech. A/Solids 20, 277–298; Terravecchia, S., 2006. Closed form in the Symmetric Boundary Element Approach. Eng. Anal. Bound. Elem. Meth. 30, 479–488].

Some examples show the goodness of the methodology proposed through a comparison with other formulations [Barenblatt, G.I., 1962. Mathematical theory of equilibrium cracks in brittle fracture. Adv. Appl. Mech. 7, 55–129; Saleh, A.L., Aliabadi, M.H., 1995. Crack growth analysis in concrete using Boundary Element Method. Eng. Fract. Mech. 51, 533–545; Aliabadi, M.H., Saleh, A.L., 2002. Fracture mechanics analysis of cracking in plain and reinforced concrete using boundary element method. Eng. Fract. Mech. 69, 267–280]. In these examples the applied loads and the length of the process zone are a priori given and kept fixed during the analysis in order to check the constitutive behavior along the process zone.

1. Introduction

The SGBEM has been utilized in the fracture mechanics only in the last decades (Li et al., 1998; Maier et al., 1993; Carini et al., 1999; Chen et al., 1999; Gray et al., 1993; Frangi, 2002; Phan et al., 2003; Salvadori, 2003). Indeed, this method is perfectly adaptable to the fracture mechanics problems both for the simplicity of the crack boundary discretization during the growth and for the employment of the symmetric and in sign defined operators.

In the ambit of the brittle materials, the most known cohesive model was proposed by Barenblatt (1962). He assumes a non-linear distribution of closing cohesive forces along the process zone of the crack having as extreme value the ultimate tensile stress σ_u at the crack tip. These external actions are used to simulate the damage of the material at the process zone.

In the ambit of the concrete materials Aliabadi and Saleh (2002), Saleh and Aliabadi (1995), Mi and Aliabadi (1994) have developed a model for crack growth by using Dual BEM. They replace the fictitious crack by closing cohesive forces acting on both crack surfaces, that is in accordance with the Barenblatt hypothesis.

In the past, many models have been developed in the ambit of the SGBEM for the analysis of a cracked body (linear and

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non-linear EFM) where in the mixed-value analysis a vector solution is obtained for the discretized body. This vector collects, as unknown quantities, the reactive forces of the constrained boundary nodes, the absolute displacements of the free boundary nodes and the displacement discontinuities of the crack boundary (Carini et al., 1999; Williams et al., 2006). This strategy is useful when the stress field is described by Westergaard or Williams functions where the SIFs (Stress Intensity Factors) are defined using Linear Elastic Fracture Mechanics methods, as the Crack Opening Displacement methods (Blandford et al., 1981; Gdoutos, 1993; Portela et al., 1992) and J-Integral approach (Rice, 1968).

The objective of this work is to obtain in the analysis phase the absolute displacements of the fictitious crack nodes and through them to evaluate the other nodal quantities distributed along the boundary: the reactive forces of the constrained boundary Γ_1, the displacements of the free boundary Γ_2 and the mutual forces of the opened one Γ_c.

For any geometry of the solid through all these quantities it is possible to evaluate the internal stress field by using any method, but it is not the aim of the present work.

The proposed model is based on the simple consideration that a cracked body is a solid that has a "particular" geometry whose analysis is obtainable through the displacement formulation employed by some of the present authors in the ambit of the Symmetric Boundary Elements Method. In this analysis it is possible to evaluate the absolute displacements of the both crack edge nodes, the crack tip included. This goal is reached in a program where all the equation system coefficients are computed in closed form.

Besides a strategy allowing the evaluation of a stiffness function along the process zone has been developed. It is based on the constitutive relations $\sigma - \Delta u$ and $\tau - \Delta w$ between the normal and tangential forces and the corresponding relative displacements. On the base of these stiffness functions, appropriately modelled on the boundary, the nodal stiffness will be evaluated by using the following strategies:

(1) In the Section 3, an internal distribution of nodal springs is included inside the cohesive zone for both normal and tangential quantities, or
(2) In the Section 4, a suitable number of substructures are inserted at the process zone of the crack, each having a different Young modulus and Poisson coefficient.

These strategies have been developed inside the software Karnak.sGbem (Panzeca et al., 2002a). In this software all the coefficients of the equation system have been computed in closed form, it permitting to obtain in the analysis phase the nodal absolute displacements of the opened opposite sides.

2. The characteristic equation of an opened cracked body

Let the homogeneous elastic two-dimensional body of domain Ω be bounded by the constrained Γ_1 and free Γ_2 boundaries, as well as by the cracked zone Γ_c, being $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_c$, as it is shown in Fig. 1a. The external actions are the forces $\bar{\mathbf{f}}$ on Γ_2, the imposed displacements $\bar{\mathbf{u}}$ on Γ_1 and the body forces $\bar{\mathbf{b}}$ in Ω. We suppose that the friction does not happen between the two crack edges.

It is known that in order to allow a structural analysis by using the Boundary Elements Method in its symmetric formulation, it is necessary the employment of both the classical S.Is. of the displacements and of the tractions, that is

$$\mathbf{u}(\mathbf{x}) = \int_\Gamma \mathbf{G}_{uu}(\mathbf{x};\mathbf{x}')\mathbf{f}(\mathbf{x}')\,d\Gamma + \int_\Gamma \mathbf{G}_{ut}(\mathbf{x};\mathbf{x}',\mathbf{n}')\mathbf{v}(\mathbf{x}')\,d\Gamma + \mathbf{u}^l(\mathbf{x}), \tag{1a}$$

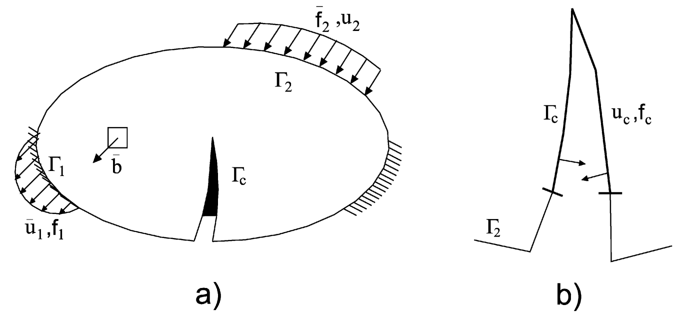

Fig. 1. a) Cracked body; b) details of the cracked zone.

$$\mathbf{t}(\mathbf{x},\mathbf{n}) = \int_\Gamma \mathbf{G}_{tu}(\mathbf{x},\mathbf{n};\mathbf{x}')\mathbf{f}(\mathbf{x}')\,d\Gamma + \int_\Gamma \mathbf{G}_{tt}(\mathbf{x},\mathbf{n};\mathbf{x}',\mathbf{n}')\mathbf{v}(\mathbf{x}')\,d\Gamma + \mathbf{t}^l(\mathbf{x},\mathbf{n}), \tag{1b}$$

where the symbology introduced in precedent works (Maier and Polizzotto, 1987; Polizzotto, 1988) has been employed. The matrix $\mathbf{G}_{hk}(\mathbf{x},\mathbf{x}')$ collects the Fundamental Solutions (F.Ss.) in which the effect at $\mathbf{x}$ is specified by the first index h, the cause at $\mathbf{x}'$ is specified by the dual quantity associated to the second index k, being $h, k = u, t, \sigma$. The vector $\mathbf{v}(\mathbf{x}') = -\mathbf{u}(\mathbf{x}')$ represents the distortion, that is the difference between the null displacement to be imposed in the boundary of the complementary domain $\Omega_\infty \backslash \Omega$ and the displacement of the real boundary of the body, when the solution is reached.

In the previous relationships the force vector $\mathbf{f}(\mathbf{x}')$ collects the unknown reactive forces on Γ_1, the known ones on Γ_2 and the unknown cohesive closing forces on the process zone Γ_c, whereas the distortion vector $\mathbf{v}(\mathbf{x}')$ collects the known absolute displacements on Γ_1, the unknown ones on Γ_2, and the unknown absolute displacements of both the edges of Γ_c, all changed in sign.

For this body we want to determine an elasticity equation connecting the quantities associated to the process zone by using a strategy based on the symmetric approach of the Boundary Elements Method.

At this aim, the classical Dirichlet and Neumann conditions must be imposed on the body boundary, i.e.

$$\mathbf{u}_1 = \bar{\mathbf{u}}_1 \quad \text{on } \Gamma_1, \tag{2a}$$

$$\mathbf{t}_2 = \bar{\mathbf{f}}_2 \quad \text{on } \Gamma_2 \tag{2b}$$

whereas the displacement $\mathbf{u}_c$ and cohesive closing force $\mathbf{f}_c$ vectors must be computed on Γ_c, i.e. in the cracked zone.

When we introduce the S.Is. of the displacements and of the tractions, the following boundary integral equations may be written:

on Γ_1

$$\int_{\Gamma_1} \mathbf{G}_{uu}\mathbf{f}_1 + \oint_{\Gamma_1} \mathbf{G}_{ut}(-\bar{\mathbf{u}}_1) + \frac{1}{2}\bar{\mathbf{u}}_1 + \int_{\Gamma_2} \mathbf{G}_{uu}\bar{\mathbf{f}}_2 + \int_{\Gamma_2} \mathbf{G}_{ut}(-\mathbf{u}_2) + \int_{\Gamma_c} \mathbf{G}_{uu}\mathbf{f}_c + \int_{\Gamma_c} \mathbf{G}_{ut}(-\mathbf{u}_c) + \int_\Omega \mathbf{G}_{uu}\bar{\mathbf{b}} = \bar{\mathbf{u}}_1; \tag{3a}$$

on Γ_2

$$\int_{\Gamma_1} \mathbf{G}_{tu}\mathbf{f}_1 + \int_{\Gamma_1} \mathbf{G}_{tt}(-\bar{\mathbf{u}}_1) + \oint_{\Gamma_2} \mathbf{G}_{tu}\bar{\mathbf{f}}_2 + \frac{1}{2}\bar{\mathbf{f}}_2 + \int_{\Gamma_2} \mathbf{G}_{tt}(-\mathbf{u}_2) + \int_{\Gamma_c} \mathbf{G}_{tu}\mathbf{f}_c + \int_{\Gamma_c} \mathbf{G}_{tt}(-\mathbf{u}_c) + \int_\Omega \mathbf{G}_{tu}\bar{\mathbf{b}} = \bar{\mathbf{f}}_2; \tag{3b}$$

on Γ_c