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Dimensional reduction of a piezoelectric composite rod

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ABSTRACT

In the present paper, a new generalized Timoshenko model is constructed for a composite rod with embedded or attached piezoelectric materials. This model is applicable to composite rods without prescribed electric potential along the lateral surfaces. The Variational-Asymptotic Method (VAM) is applied as a mathematical tool to carry out the dimensional reduction process. The present reduced model captured the effects of dielectric as well as the polarization of the piezoelectric material, which justifies its coupled electromechanical nature. First, the three-dimensional electromechanical enthalpy is asymptotically approximated by VAM using the slenderness of the rod as the small parameter and subsequently an equivalent one-dimensional electromechanical enthalpy is developed. Energy terms, which are asymptotically correct up to the second order are kept in the approximate enthalpy expression. For engineering applications, the approximate enthalpy is then transformed into a generalized Timoshenko model which has the traditional six mechanical degrees of freedom along with an extra one-dimensional electric degree of freedom.

1. Introduction

Since their discovery by the Curie brothers (Katzir, 2003), piezoelectric materials have been applied successfully in numerous scientific fields. Ultrasonic technology, MEMS/NEMS industry, micro acoustic generator, miniaturized electronic transformer, and smart structures are among the well-known application areas of piezoelectric materials. What distinguishes a piezoelectric material is its remarkable property to create a conversion interface between two forms of energy, i.e., mechanical to electric or vice versa. This two-way coupling capability, along with other properties, such as rapid response, high operating bandwidth and low power consumption, make piezoelectric materials suitable for use both as sensors and actuators (Aldraihem and Khdeir, 2003).

Piezoelectric beam actuators and sensors are very common in scientific applications. Rosen type piezoelectric transformer (Rosen, 1956) is a perfect example where piezoelectric material is used to build a beam like structure. Inspite of ever increasing sophistication in modern three-dimensional (3D) computational techniques, sometime, it is neither computationally feasible nor desirable to make a full 3D analysis in an electromechanically coupled framework. As an alternative, researchers try to exploit the slender nature of beam like structures and simplify the analysis using one-dimensional (1D) reduced models. So, a natural question always remain: can these reduced models properly capture the electromechanical effect?

Most of the beam-modeling techniques in the existing literature use conventional displacement field based approaches, where the deformation patterns of a structure are assumed at the very beginning of the analysis (Crawley and Anderson, 1990; Robbins and Reddy, 1991; Park et al., 1996; Zhang and Sun, 1996; Smyser and Chandrashekhara, 1997; Saravanos and Heyliger, 1995; Reddy and Cheng, 2001). Sometimes, these models become oversimplified due to these a priori assumptions, for example, existence of uniaxial stress state, plane strain state etc. Also, it is very difficult, if not impossible, to assume correct deformational pattern to capture the physics of electromechanical coupling. These assumption based models work reasonably well for uncoupled problems, but their applicability and authenticity in a coupled framework always remain questionable. A comprehensive review on piezoelectric beam like transformer modeling can be obtained in Yang (2007).

Recently, the Variational Asymptotic Method (VAM) (Berdichevsky, 1979) has been applied to model smart beams made of piezoelectric material. This method has both merits of variational methods (*viz.*, systematic and easily implemented numerically) and asymptotic methods (*viz.*, without *ad hoc* kinematic assumptions). In the past, VAM has been successfully applied to model composite beams (Yu et al., 2002). Cesnik et al. used VAM to model smart beams made of piezoelectric fiber composites. They also developed one-way coupled classical model for smart thin-walled beams (Cesnik and Shin, 2001), smart solid beams (Cesnik and Ortega-Morales, 2001), and a coupled refined model for smart

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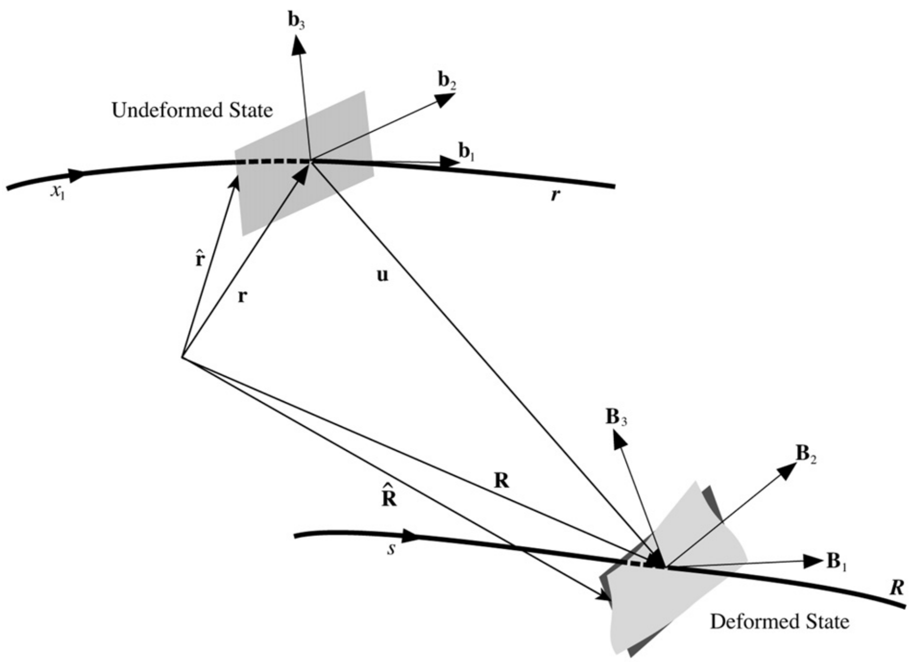

Fig. 1. Schematic of beam deformation.

beams (Cesnik and Palacios, 2005). In the refined model, they applied a Ritz-type assumed mode approximation to accommodate the finite size effect of the cross section. Very recently, the authors have developed a simpler theory for applying variational asymptotic method in the modeling of piezoelectric composite beams (Roy et al., 2007). The work was based on the previous work on the modeling of composite beams (Yu, 2002) and piezoelectric beams (Le, 1999).

In most of the reported studies on piezoelectric beams, it is assumed that the lateral surfaces are fully or partially electroded with prescribed potentials. The present work attempts to study the complementary cases where there is no prescribed electric potential on the lateral surfaces. Thus, the focus of this study is to use VAM to rigorously reduce the dimension of such piezoelectric composite rods to develop a reduced 1D beam model. The constructed model and the accompanying numerical code, can have significant application potential in the MEMS/NEMS industry, acoustic field, smart structures, where beam like actuators and sensors made of piezoelectric composites are frequently used (Yang, 2007).

1.1. Three-dimensional formulation

The Hamilton's principle governing the 3D behavior of a piezoelectric rod is stated as

$$\int_{t_1}^{t_2} \left[\delta(\mathcal{K}-\mathcal{H})+\overline{\delta\mathcal{W}}\right]dt=0 \tag{1}$$

where t_1 and t_2 are arbitrary fixed times, $\mathcal{K}$ and $\mathcal{H}$ are the kinetic and electric enthalpy, respectively, and $\overline{\delta\mathcal{W}}$ is the virtual work of applied loads and electric charges (if exist). The bar is used to indicate that the virtual work needs not to be the exact variations of functionals. The electric enthalpy of piezoelectric material is:

$$\mathcal{H}=\frac{1}{2}\int_{\mathcal{V}}\left(\boldsymbol{\Gamma}^T:\boldsymbol{C}^{\boldsymbol{E}}:\boldsymbol{\Gamma}-2\boldsymbol{E}\cdot\boldsymbol{e}:\boldsymbol{\Gamma}-\boldsymbol{E}^T\cdot\boldsymbol{\varepsilon}^{\boldsymbol{\Gamma}}\cdot\boldsymbol{E}\right)d\mathcal{V} \tag{2}$$

where $\boldsymbol{C}^E$ is the elastic tensor at constant electric field, $\boldsymbol{\Gamma}$ is the strain tensor, $\boldsymbol{e}$ is the piezoelectric tensor, $\boldsymbol{E}$ is the electric field vector, $\boldsymbol{\varepsilon}^{\Gamma}$ is the dielectric tensor at constant strain field, and $\mathcal{V}$ is the space occupied by the structure. It is noted that although the focused application is piezoelectric rods, the present formulation is equally applicable to smart rods made of other smart materials characterized by a constitutive model with the same mathematical structure as Eq. (2).

In Fig. 1, a beam is represented by a reference line r measured by coordinate x_1. A typical cross section s with h as its characteristic dimension is described by cross-sectional Cartesian coordinates x_α (here and throughout the paper, Greek indices $\alpha, \beta \ldots$ assume values 2 and 3 while Latin indices assume 1, 2, and 3. Repeated indices are summed over their ranges except where explicitly indicated). At each point along r, an orthonormal triad $\mathbf{b}_i$ is introduced such that $\mathbf{b}_i$ is tangent to the coordinate curve x_i. The position vector $\hat{\mathbf{r}}$ of an arbitrary point in the undeformed structure is given by

$$\hat{\mathbf{r}}(x_1,x_2,x_3)=\mathbf{r}(x_1)+x_\alpha\mathbf{b}_\alpha \tag{3}$$

where $\mathbf{r}$ is the position vector of a point on the reference line and $\mathbf{r}'=\mathbf{b}_1$. Here $(\,)'$ denotes the partial derivative with respect to x_1. When the beam deforms, the triad $\mathbf{b}_i$ rotates to coincide with a new triad $\mathbf{B}_i$. Here $\mathbf{B}_1$ is normal to the unwarped cross section, but not tangent to the beam deformed reference line due to transverse shear deformation. For the convenience of derivation, we introduce another triad $\mathbf{T}_i$ associated with the deformed beam (see Fig. 2), with $\mathbf{T}_1$ tangent to the deformed beam reference line and $\mathbf{T}_\alpha$ is determined by a rotation about $\mathbf{T}_1$. The difference in the orientations of $\mathbf{T}_i$ and $\mathbf{B}_i$ is due to small rotations associated with transverse shear deformations, as shown in Fig. 2. In Fig. 2, $2\gamma_{13}$ is a small angle in 1–3 plane caused by the shear deformation while another rotation due to $2\gamma_{12}$ is not sketched for clarity.

The material points having position vector $\hat{\mathbf{r}}$ in the undeformed beam can be located after deformation by the vector function given by

$$\hat{\mathbf{R}}(x_1,x_2,x_3)=\mathbf{R}(x_1)+x_\alpha\mathbf{T}_\alpha(x_1)+w_i(x_1,x_2,x_3)\mathbf{T}_i(x_1) \tag{4}$$

where $\mathbf{R}$ is the position vector to a point on the reference line of the deformed beam and defined as the average of $\hat{\mathbf{R}}(x_1,x_2,x_3)$ over the reference cross section. In Eq. (4), w_1 denotes the out of plane warping while w_2 and w_3 denote the inplane components