European Journal of Mechanics A/Solids 44 (2014) 104–115

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

Free vibration analysis of functionally graded carbon nanotube-reinforced composite cylindrical panel embedded in piezoelectric layers by using theory of elasticity

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ARTICLE INFO

Article history:
Received 2 April 2013
Accepted 5 October 2013
Available online 15 October 2013

Keywords:
Vibration
CNTRC cylindrical panel
Piezoelectric

ABSTRACT

In this paper free vibration behavior of functionally graded carbon nanotube-reinforced composite (FG-CNTRC) cylindrical panel embedded in piezoelectric layers with simply supported boundary conditions is investigated by using three-dimensional theory of elasticity. By using Fourier series expansion along the longitudinal and latitudinal directions and state space technique across the thickness direction, state space differential equations are solved analytically. The traction-free surface conditions then give rise to the characteristic equation for natural frequencies. Accuracy and convergence of the present approach are validated by comparing the numerical results with those found in literature. In addition, the effects of volume fraction of CNT, four cases of FG-CNTRC, piezoelectric layer thickness, mid radius to thickness ration and modes number on the vibration behavior of the hybrid cylindrical panel are also examined.

1. Introduction

The high strength and stiffness of carbon nanotube (CNT) make it as a very promising reinforcing constituent as compared to the conventional fibers currently used in composite structures such as beams, plates and shells. Introduction of CNTs into polymer matrix increases the application of reinforcing composite elements. On the other hand, piezoelectric materials have coupled effects between the electric fields and the elastic deformation. It is possible to make a system of intelligent materials by combining these piezoelectric materials with nanocomposites. The piezoelectric FG-CNTRC plates are composed of nanocomposite plate imbedded in piezoelectric layers. Study on the mechanical and thermal properties of CNTRC structures has increased by many researchers in recent years. Thostenson et al. (2001) presented a review on the researches and application of CNT and CNTRC. Gou et al. (2004) used the molecular dynamics (MD) simulations and experimental method to investigate the interfacial bonding of single-walled nanotube (SWNT) reinforced epoxy composites. Wuite and Adali (2005) carried out a multi-scale analysis of the deflection and stress behavior of CNT-reinforced polymer composite beams. Vodenitcharova and Zhang (2006) investigated pure bending and bending-induced local buckling of a nanocomposite beam reinforced by an SWNT computationally as well as experimentally using Airy stress-function approach. Qiu et al. (2007) introduced an effective infiltration-based vacuum-assisted resin transfer moulding method to fabricate multifunctional multi-scale composites. Qiu et al. (2009) used a modified finitely extensible nonlinear elastic dumbbell model to discuss the concentration-dependent and shear-rate-dependent viscosity of CNTs dispersed polymer solution. Shen (2009) discussed nonlinear bending behavior of simply supported, functionally graded composite plates reinforced by SWCNTs subjected to transverse uniform or sinusoidal load in thermal environments. Qiu and Wang (2010) examined the reaction kinetics of CNT-reinforced epoxy resin. Formica et al. (2010) used an equivalent continuum model and Eshelby–Mori–Tanaka approach to study the vibrational properties of CNTRC. Shen and Zhang (2010) used multi-scale approach to discuss thermal buckling and post buckling behavior of functionally graded nanocomposite plates reinforced by SWCNTs subjected to in-plane temperature variation. Based on Timoshenko beam theory and von Karman geometric nonlinearity, Ke et al. (2010) discussed nonlinear free vibration of FG nanocomposite beams reinforced by SWCNTs by using Ritz method. Shen (2011a,b) used higher order shear deformation theory as well as a von Kármán-type of kinematic nonlinearity to investigate the postbuckling behavior of nanocomposite cylindrical shells reinforced by SWCNTs and subjected to axial environments. Based on a micromechanical model and multi-scale approach, Shen (2011a,b) discussed post buckling behavior of FG-CNTRC cylindrical shells subjected to mechanical load in

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http://dx.doi.org/10.1016/j.euromechsol.2013.10.002

Nomenclature

$C_{ij}(i,j = 1,2,\ldots,6)$	material elastic constants
D_r, D_θ and D_z	electric displacement components
E_r, E_θ and E_z	electric field components in r, θ and z direction
$E_{11}^{\mathrm{CN}}, E_{22}^{\mathrm{CN}}, G_{12}^{\mathrm{CN}}, E^{\mathrm{m}}, G^{\mathrm{m}}$	Young's modulus, and shear modulus of carbon nanotube and matrix, respectively
$\eta_i(i = 1,2,3)$	scale factors of CNT
V_{CN}, V_{m}	volume fractions of carbon nanotube and matrix, respectively
W_{CN}, ρ_{CN}	mass fraction and density fraction of CNT, respectively
ρ_{m}	density fraction of matrix
$\sigma_i(i = r,\theta,z)$	normal stresses
$\tau_{r\theta}\ \tau_{\theta z}\ \tau_{rz}$	shear stresses
e	piezoelectric constant
δ	state vector
$\beta_i(i = 1,2,3)$	dielectric constants
ϕ	electric potential
h_{c}, h_{p}	thicknesses of the nanocomposite and piezoelectric layers
n, m	half wave numbers in the z- and θ- directions
u_r,u_θ,u_z	displacements in the r-, θ- and z- direction, respectively
$\gamma_{r\theta},\gamma_{zr},\gamma_{\theta z}$	shear strains
$\varepsilon_i(i = r,\theta,z)$	normal strains
δ_{c}, δ_{p}	statevectors of the elastic and piezoelectric layers
ψ	electric voltage

thermal environments. Based on a higher order shear deformation plate theory, Wanga and Shen (2011) used an improved perturbation technique to investigate the nonlinear vibration of FG-SWCNT plates rested on elastic foundation in thermal environments. Zhang et al. (2012) introduced porous multi-walled CNT/polyaniline to control electrical and thermal conductivity of nanocomposite. Based on first-order shear deformation (FSDT), Mehrabadi et al. (2012) discussed mechanical buckling behavior of FG nanocomposite plate reinforced by SWCNTs by using Mindlin plate theory. Zhu et al. (2012) carried out bending and free vibration analysis of composite plates reinforced by SWCNTs by using the finite element method and the first-order shear deformation plate theory. Wang and Shen (2012) investigated nonlinear bending and vibration behavior of sandwich plate with CNTRC face sheets by making the use of using multi-scale approach and two-step perturbation technique. Yas and Heshmati (2012) used Timoshenko beam theory to discuss vibration behavior of FG nanocomposite beams reinforced by randomly oriented straight SWCNTs and subjected to moving load. Alibeigloo (2013) presented an analytical solution for bending behavior of FG-CNT composite plate integrated with piezoelectric actuator and sensor subjected to uniform mechanical load. Bhardwaj et al. (2013) investigated the nonlinear static and dynamic behavior of cross-ply CNTRC laminated plate by using the double Chebyshev series. By using higher order shear deformation theory and Von Karman type of kinematic nonlinearity, Shen (2012) discussed postbucking of FG-CNTRC cylindrical panel in thermal environment. Nonlinear vibration of FG-CNTRC cylindrical shell was investigated by Shen and Xiang (2012) using higher order shear deformation theory with a Von Karman type of kinematic nonlinearity. Moradi-Dastjerdi et al. (2013) analyzed the dynamic behavior of FG-CNTRC cylindrical shell subjected to impact load by making the use of mesh free method. By using Eshelby–Mori–Tanaka approach and two-dimensional differential quadrature method, free vibration analysis of CNTRC cylindrical panel was presented by Sobhani Aragh et al. (2012). To the authors' knowledge, however, three-dimensional free vibration analysis of hybrid FG-CNTRC cylindrical panel with surface piezoelectric layers has not been yet reported. In this paper we used theory of linear elasticity to investigate vibrational behavior of simply supported hybrid FG-CNTRC cylindrical panel. In this investigation, it is assumed that the bonding interface between the piezoelectric and FG-CNTRC layers are solid and the transverse stress and displacement components are continuous. It is noted that the result of this benchmark solution can be used to assess the validity of assumption used in conventional dimensional theory.

2. Basic equations

2.1. FG-CNTRC layer

Consider a CNTRC cylindrical panel with geometry and dimensions according to Fig. 1. The CNT reinforcement is either uniformly distributed (UD) or functionally graded (FG) in four cases; FG − ∇, FG−Δ, FG−X and FG − ◇ in the radial direction. Displacements component along the r, θ and z directions are denoted by u_r,u_θ and u_z, respectively. According to the rule of mixture and considering the CNT efficiency parameters, the effective material properties of mixture of CNTs and isotropic polymer matrix can be written as follows [6]

$$E_{11} = \eta_1 V_{\mathrm{CN}} E_{11}^{\mathrm{CN}} + V_{\mathrm{m}} E^{\mathrm{m}} \tag{1.1}$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{\mathrm{CN}}}{E_{22}^{\mathrm{CN}}} + \frac{V_{\mathrm{m}}}{E^{\mathrm{m}}} \tag{1.2}$$

$$\frac{\eta_3}{G_{12}} = \frac{V_{\mathrm{CN}}}{G_{12}^{\mathrm{CN}}} + \frac{V_{\mathrm{m}}}{G^{\mathrm{m}}} \tag{1.3}$$

Volume fraction of CNT and matrix constituents are related as

$$V_{\mathrm{CN}} + V_{\mathrm{m}} = 1 \tag{2}$$

The CNT volume fraction for the five cases of UD, FG − ∇, FG−Δ and FG−X distribution along the thickness of the panel according to Fig. 1 has the following relations, respectively

$$V_{\mathrm{CNT}} = V_{\mathrm{CNT}}^{*} \tag{3.1}$$

$$V_{\mathrm{CNT}} = 2\left(\frac{r-R}{h} + 0.5\right) V_{\mathrm{CNT}}^{*} \tag{3.2}$$

$$V_{\mathrm{CNT}} = 2\left(-\frac{r-R}{h} + 0.5\right) V_{\mathrm{CNT}}^{*} \tag{3.3}$$