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A parametric study on the buckling of functionally graded material plates with internal discontinuities using the partition of unity method

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ABSTRACT

In this paper, the effect of local defects, viz., cracks and cutouts on the buckling behaviour of functionally graded material plates subjected to mechanical and thermal load is numerically studied. The internal discontinuities, viz., cracks and cutouts are represented independent of the mesh within the framework of the extended finite element method and an enriched shear flexible 4-noded quadrilateral element is used for the spatial discretization. The properties are assumed to vary only in the thickness direction and the effective properties are estimated using the Mori-Tanaka homogenization scheme. The plate kinematics is based on the first order shear deformation theory. The influence of various parameters, viz., the crack length and its location, the cutout radius and its position, the plate aspect ratio and the plate thickness on the critical buckling load is studied. The effect of various boundary conditions is also studied. The numerical results obtained reveal that the critical buckling load decreases with increase in the crack length, the cutout radius and the material gradient index. This is attributed to the degradation in the stiffness either due to the presence of local defects or due to the change in the material composition.

1. Introduction

The functionally graded materials (FGMs) are new class of engineered materials characterized by *smooth and continuous transition* in properties from one surface to another (Koizumi, 1993). As a result, the FGMs are macroscopically homogeneous and are preferred over the laminated composites for structural integrity. The tunable thermo-mechanical property of the FGM has attracted researchers to study the static and the dynamic behaviour of structures made of FGM under mechanical (Zenkour, 2006, 2007; Reddy, 2000; Singha et al., 2011) and thermal loading (Natarajan et al., 2011a; Praveen and Reddy, 1998; Dai et al., 2011; Ganapathi and Prakash, 2006; Janghorbana and Zare, 2011; Zenkour and Mashat, 2010; Zhao et al., 2009a). Praveen and Reddy (1998) and Reddy and Chin (2007) studied the thermo-elastic response of ceramic-metal plates using first order shear deformation theory (FSDT) coupled with 3D heat conduction equation. Their study concluded that the structures made up of FGM with ceramic rich side exposed to elevated temperatures are susceptible to buckling due to the through thickness temperature variation. The buckling of skewed FGM plates under mechanical and thermal loads were studied in Ganapathi and Prakash (2006), Ganapathi et al. (2006) employing the FSDT and by using the shear flexible quadrilateral element. Efforts have also been made to study the mechanical behaviour of FGM plates with geometrical imperfection (Shariat and Eslami, 2006). Saji et al. (2008) studied thermal buckling of FGM plates with material properties dependent on both the composition and temperature. They found that the critical buckling temperature decreases when material properties are considered to be a function of temperature. Ganapathi and Prakash (2006) studied the buckling of FGM skewed plate under thermal loading. FGM plates or in general plate structures, may develop flaws during manufacturing or after they have been put into service. Hence it is important to understand the response of a FGM plate with an internal flaw. It is known that cracks or local defects affect the response of a structural member. This is because, the presence of the crack introduces local flexibility and anisotropy. The vibration

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of cracked FGM structures are fairly dealt in the literature. Kitipornchai et al. (2009) studied non-linear vibration of edge cracked functionally graded Timoshenko beams using Ritz method. Yang et al. (2010) studied the non-linear dynamic response of a functionally graded plate with a through-width crack based on Reddy's third-order shear deformation theory. Dolbow and Gosz (2002) employed the extended finite element method (XFEM) to compute mixed mode stress intensity factors for a crack in a functionally graded material. Natarajan et al. (2011b), Natarajan (2011) studied the influence of cracks on the vibration and mechanical buckling of functionally graded material plates. The above list is no way comprehensive and interested readers are referred to the literature and references therein and a recent review paper by Jha et al. (2013) on FGM plates. To the author's knowledge, the influence of the presence of an internal flaw, viz., cracks and cutouts on the critical buckling load and critical buckling temperature has not been studied earlier.

In this paper, we study the buckling behaviour of FGM plates with local defects, viz., cracks and cutouts. In this study, cracks and cutouts are considered as internal flaw. A structured quadrilateral mesh is employed and the local defects are represented independent of the underlying finite element (FE) mesh by enriching the FE approximation basis with additional functions. An enriched shear flexible 4-noded element proposed in Natarajan et al. (2011b), Natarajan (2011) is used for this study. The influence of various geometric parameters, viz., the plate aspect ratio, the thickness of the plate, the crack length, the crack orientation and location, the cutout radius, the support conditions and the gradient index on the critical buckling load is numerically studied.

The paper is organized as follows, the next section will give a brief over of Reissner–Mindlin plate theory and an introduction to FGM. Section 3 discusses the spatial discretization within XFEM framework and numerical integration over enriched elements. Section 4 presents results for the buckling analyses of FGM plates with geometric defects (cracks) and material discontinuity (cutouts), followed by concluding remarks in the last section.

2. Theoretical formulation

2.1. Functionally graded material

A functionally graded material (FGM) rectangular plate (length a, width b and thickness h), made by mixing two distinct material phases: a metal and ceramic is considered with coordinates x,y along the in-plane directions and z along the thickness direction (see Fig. 1). The material on the top surface ($z = h/2$) of the plate is ceramic and is graded to metal at the bottom surface of the plate ($z = -h/2$) by a power law distribution. The homogenized material properties are computed using the Mori-Tanaka Scheme (Benvensite, 1987; Reddy, 2000; Qian et al., 2004).

2.1.1. Estimation of mechanical and thermal properties

Based on the Mori-Tanaka homogenization method, the effective bulk modulus K and shear modulus G of the FGM are evaluated as (Benvensite, 1987; Qian et al., 2004)

$$\frac{K-K_m}{K_c-K_m} = \frac{V_c}{1+(1-V_c)\frac{3(K_c-K_m)}{3K_m+4G_m}}$$
$$\frac{G-G_m}{G_c-G_m} = \frac{V_c}{1+(1-V_c)\frac{(G_c-G_m)}{G_m+f_1}} \tag{1}$$

where

$$f_1 = \frac{G_m(9K_m+8G_m)}{6(K_m+2G_m)} \tag{2}$$

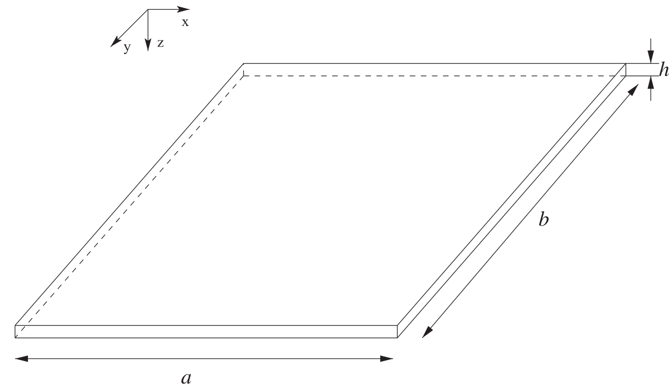

Fig. 1. Coordinate system of a rectangular FGM plate.

Here, V_i ($i = c,m$) is the volume fraction of the phase material. The subscripts c and m refer to the ceramic and metal phases, respectively. The volume fractions of the ceramic and metal phases are related by $V_c + V_m = 1$, and V_c is expressed as

$$V_c(z) = \left(\frac{2z+h}{2h}\right)^n, \quad n \geq 0 \tag{3}$$

where n in Equation (3) is the volume fraction exponent, also referred to as the material gradient index. Fig. 2 shows the variation of the volume fractions of ceramic and metal, respectively, in the thickness direction z for the FGM plate. The top surface is ceramic rich and the bottom surface is metal rich. The effective Young's modulus E and Poisson's ratio ν can be computed from the following expressions:

$$E = \frac{9KG}{3K+G} \nu = \frac{3K-2G}{2(3K+G)} \tag{4}$$

The effective mass density ρ is given by the rule of mixtures as $\rho = \rho_c V_c + \rho_m V_m$. The effective heat conductivity $\kappa_{\rm eff}$ and the coefficient of thermal expansion $\alpha_{\rm eff}$ is given by:

$$\frac{\kappa_{\rm eff}-\kappa_m}{\kappa_c-\kappa_m} = \frac{V_c}{1+V_m\frac{(\kappa_c-\kappa_m)}{3\kappa_m}}$$
$$\frac{\alpha_{\rm eff}-\alpha_m}{\alpha_c-\alpha_m} = \frac{\left(\frac{1}{\kappa_{\rm eff}}-\frac{1}{\kappa_m}\right)}{\left(\frac{1}{\kappa_c}-\frac{1}{\kappa_m}\right)} \tag{5}$$

2.1.2. Temperature distribution through the thickness

The temperature variation is assumed to occur in the thickness direction only and the temperature field is considered to be constant in the xy -plane. In such a case, the temperature distribution along the thickness can be obtained by solving a steady state heat transfer equation

$$-\frac{d}{dz}\left[\kappa(z)\frac{dT}{dz}\right] = 0, \quad T = T_c \text{ at } z = h/2; T = T_m \text{ at } z = -h/2 \tag{6}$$

The solution of Equation (6) is obtained by means of a polynomial series (Wu, 2004) as

$$T(z) = T_m + (T_c - T_m)\eta(z,h) \tag{7}$$

where,