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Coupled thermoelastic effect in free vibration analysis of anisotropic multilayered plates and FGM plates by using a variable-kinematics Ritz formulation

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ABSTRACT

A fully coupled thermoelastic formulation is developed to deal with free vibration analysis of anisotropic composite plates and isotropic/sandwich FGM plates. The proposed formulation is developed by combining refined hierarchical plate models and a trigonometric Ritz method. The temperature is considered as a primary variable and allows the evaluation of the temperature field effects in the free vibration analysis. The temperature profile across the plate thickness is always modeled with a layer-wise kinematics description, nevertheless both equivalent single layer and layer-wise approaches are properly and effectively used for the displacement variables. In the 2D and quasi-3D higher-order variable-kinematics plate theories, each displacement variable, in the displacement field, is treated independently from the others. Such artifice allows to select scrupulously each expansion order for each primary variable regarding to the required accuracy and the computational cost. So-called Ritz fundamental primary nuclei related to the coupled thermal and mechanical fields are generated by virtue of an unconventional principle of virtual displacement accounting for the internal thermal virtual work to reproduce the coupling effect. Each fundamental primary nucleus is mathematically invariant with respect to the used kinematics description, the employed expansion orders and the chosen Ritz functions. The thermoelastic coupling is investigated in terms of natural frequencies and the effect of stacking sequence and length-to-thickness ratio for lower and higher modes is discussed.

1. Introduction

The theory of thermoelasticity represents a generalization of both the theory of elasticity and the theory of heat conduction in solids. It is a branch of applied mechanics that is concerned with the effects of heat on the deformation and stresses of solid bodies, which are considered to be elastic. It is therefore considered as an extension of the conventional theory of isothermal elasticity to those processes in which deformation and stresses are produced not only by mechanical forces, but also by temperature variations. The effect of the temperature field on the deformation field is not a one-way phenomenon (Nowinski, 1978), a deformation of the body produce temperature variation, therefore the mechanical and thermal aspects are inseparable. By comparison with the history of the theory of elasticity which is traced back to Galileo Galilei in the 16th century (Galilei, 1638), the history of thermoelasticity is much younger. Coupling between deformation and temperature fields was originally postulated by Duhamel (1837). In his work the formulation of boundary values problem and the equations for the coupling of the temperature field and the body's deformation were obtained. Thermoelasticity equations, as highlighted by Hetnarski and Elsami (2007), were proposed by Neumann (Neumann, 1885) in 1885, by E. Almansi (Almansi, 1897) in 1897, by O. Tedano (Tedone, 1906) in 1906 and W. Voigt (Voigt, 1910) in 1910. In literature, a small amount of work has been devoted to the coupled thermo-mechanical analysis of structures (both thermoelastic and thermo-plastic analysis), and only few of them give numerical results. Altay and Dökmeci (1996a) have described the physical behavior of thermoelastic continuum by using advanced variational principles formulated introducing the dislocation potentials and the Lagrange undetermined multipliers to take into account the internal surface of discontinuity. The same authors provided the description of the physical behavior of a thermopiezoelectric medium in Altay and Dökmeci (1996b) by simply adding the charge equation of electrostatics in the divergence equations and the electric field potential relations in the gradient equations. Das et al. (1983) have avoided

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the use of the thermoelastic potential to solve the general problem of one-dimensional linearized simultaneous equations of thermoelasticity. Displacement and thermal fields have been obtained in the Laplace transformation domain. Cannarozzi and Umbertini (2001) have proposed a variational method for linear coupled quasi-static thermoelastic analysis. The variational support is a statement in terms of displacement, temperature, stress and heat flux. The statement has been based on the hybrid stress formulation for the elastic part and on the mixed flux-temperature formulation for the thermal part, and it has included the rate dependent terms of the energy balance equations and the initial conditions. Thermal balance and initial conditions have been weakly enforced using temperature as a Lagrange multiplier, and the thermoelastic dissipation term has been expressed via the constitutive equations, in terms of stress and temperature rates. The local displacement, temperature, stress and heat flux errors have been measured in time when temperature and/or displacement have been applied. Comparisons between coupled and uncoupled analysis, and the accuracy and efficiency of the coupled theory have been demonstrated in Chao and Oh (2004). A higher-order zig-zag plate theory (see (Carrera, 2003a) for an exhaustive overview on zig-zag plate/shell models) has been developed to refine the prediction of the fully coupled mechanical, thermal, and electric behavior. Both in-plane displacement and temperature fields, through the thickness have been constructed by superimposing a linear zig-zag field on to the smooth, globally cubic varying field. The given theory is suitable for the predictions of fully coupled behavior of thick, smart composite plates under combined mechanical, thermal, and electric loadings. The same authors have extended the proposed analysis to a three-node triangular finite element in Oh and Chao (2004). Ibrahimbegovic et al. (2005) have presented a thermo-mechanical coupling model for folded plates or non-smooth shells which can be used for the analysis of the fire resistance of cellular structures. Thermo-mechanical coupling has been considered, including radiative exchanges and an operator split solution procedure with different time steps. The motivation for the work was the development of predictive models that would be capable of describing the inelastic behavior of cellular structures, build either of folded plates and/or non-smooth shells, under sustained long term high-temperature effects. In Lee (2006), Lee has fully discussed the thermoelasticity problem of multilayered adiabatic and clamped hollow cylinders whose boundaries are subjected to time-dependent temperatures. Solutions for the temperature, displacement and thermal stress distributions have been obtained in both a transient and steady state. The method has developed stable solutions at a specific time. Tanaka et al. (1995) have proposed a new boundary element method for the analysis of quasi-static problems in coupled thermoelasticity. Through some mathematical manipulations of the Navier equation in elasticity, the heat conduction equation has been transformed into a simpler form, similar to the uncoupled-type equation with the modified thermal conductivity which shows the coupling effects. This procedure has made it possible to treat the coupled thermoelastic problem as an uncoupled one. Daneshjoo and Ramezani (2004, 2002) have proposed a new mixed finite element formulation to analyze transient coupled thermoelastic problems. Two simply supported plates, subjected to half-sine mechanical and thermal loads, have been considered. When a mechanical load is applied, the differences between the coupled and uncoupled analysis are minimum, in terms of stresses. Concerning, the relatively new Functional Graded Material (FGM) structures the applications of coupled thermoelastic formulations have already been developed. In particular, an exact solution for thermoelastic deformations of functionally graded thick rectangular plates has been proposed by Vel and Batra (2002). The same authors dealt with generalized thermoelastic deformation of laminated anisotropic thick plates (Vel and Batra, 2001). Three-dimensional thermoelastic deformations of functionally graded elliptic plates have been tackled by Cheng and Batra (1999). More generally FGM structures have been investigated in problems involving cracks and inclusions (Natarajan et al., 2011), harsh thermal environments (Nguyen-Xuan et al., 2011) and FEM applications (Sundararajan et al., 2005). The proposed formulation is based on an unconventional variational principle, which can be identified as an extension of the principle of virtual displacement (PVD) including the internal thermal virtual work. After combining the advanced variable-kinematics theories, equivalent single layer (ESL) and layer-wise (LW) (Fazzolari and Carrera, 2011, 2013a; Demasi, 2008), with a trigonometric Ritz method (Fazzolari and Carrera, 2011, 2013a, 2013b, 2013c), the Ritz fundamental primary stiffness, mass and thermal stiffness nuclei are obtained. They are composed by fundamental secondary nuclei which are expanded according to both the expansion orders adopted in the thickness functions and the number of terms in the Ritz functions expansion (Fazzolari and Carrera, 2013a). In Brischetto and Carrera (2011, 2010) some results and reliable benchmarks have been provided for one and two layered isotropic plates by using a Navier-type closed form solution, nevertheless hitherto no results are present in literature in coupled thermoelastic free vibration analysis of anisotropic composite plates and FGM plates. As FGMs were introduced and proposed with the aim to provide a new typology of material able to work in harsh temperature environment, the proposed thermoelastic formulation considering the temperature a primary variable will give a refined description of the free vibration behavior of FGM plates with assigned through-the-thickness temperature rise. Inasmuch as no results are present in the literature as regard both anisotropic layered plates and isotropic/sandwich FGM plates then the purpose of this work is to fill this gap providing assessments and numerical benchmarks of different refined theories and discuss the main insights and understandings.

2. Preliminaries

The plate geometry characteristics are shown in Fig. 1. A laminated plate composed of N_l layers is considered. The integer k, used as superscript or subscript, denotes the layer number which starts from the plate bottom. The layer geometry is denoted by the same symbols as those used for the whole multilayered plate and vice-versa. With x and y the plate middle surface Ω_k coordinates are indicated. Γ_k is the layer boundary on Ω_k. z and z_k are the plate and layer thickness coordinates; h and h_k denote the plate and layer thicknesses, respectively. $\zeta_k = 2z_k/h_k$ is the non-dimensioned local plate-coordinate; A_k denotes the k-layer thickness domain. Symbols that are not affected by the k subscript/superscripts refer to the whole plate. In the case of thermo-mechanical problems, the primary variables are the displacement vector in Eq. (1), and the scalar sovra-temperature θ^k, ($\theta = T_1 - T_0$, temperature T_1 is referred to the external room temperature T_0). Due to the high rate of the spatial temperature gradient the variable θ is always modeled with a layer-wise kinematics description (see Eq. (20)).

$$\mathbf{u}^k = \begin{bmatrix} u_x^k & u_y^k & u_z^k \end{bmatrix}^T \tag{1}$$

Superscript T represents the transposition operator. The stress, $\boldsymbol{\sigma}$, and the strain, $\boldsymbol{\varepsilon}$, are expressed as follows:

$$\begin{aligned}
\boldsymbol{\sigma}_{pG}^k &= \begin{bmatrix} \sigma_{xx}^k & \sigma_{yy}^k & \tau_{xy}^k \end{bmatrix}^T, \quad \boldsymbol{\varepsilon}_{pG}^k = \begin{bmatrix} \varepsilon_{xx}^k & \varepsilon_{yy}^k & \gamma_{xy}^k \end{bmatrix}^T \\
\boldsymbol{\sigma}_{nG}^k &= \begin{bmatrix} \tau_{xz}^k & \tau_{yz}^k & \sigma_{zz}^k \end{bmatrix}^T, \quad \boldsymbol{\varepsilon}_{nG}^k = \begin{bmatrix} \gamma_{xz}^k & \gamma_{yz}^k & \varepsilon_{zz}^k \end{bmatrix}^T
\end{aligned} \tag{2}$$