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Shakedown analysis of 90-degree mitred pipe bends

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ABSTRACT

The behaviours of smooth 90-degree pipe bends under cyclic loading have received substantial attention in recent years where shakedown and ratchetting domains have been determined. However, such data are considerably lacking for mitred pipe bends. In the current research, the lower bound shakedown limit loads of 90-degree mitred pipe bends are determined via a simplified direct non-cyclic numerical technique recently developed by Abdalla et al. (2007) The analysed mitred pipe bends are subjected to the combined effect of steady internal pressures and cyclic in-plane or out-of-plane bending moments. Both in-plane closing and opening bending moment cases are considered. The shakedown boundaries of three mitred pipe bend geometries with one, two, and three welded joints are determined and compared with the shakedown boundary of a smooth 90-degree pipe bend. All analysed bends have diameter to thickness ratio of 25 and bend radius of 1.5 times the pipe mean diameter. The results indicate that the shakedown boundaries of mitred bends have reduced domains compared with the smooth pipe bend of similar geometrical parameters. Shakedown domains of mitred bends increase in size as the number of welded joints increase until it approaches the shakedown boundary of the smooth pipe bend simulating a mitred bend with infinite number of welded joints. The percentage of the area under shakedown domain for the mitred pipe bends to that of the smooth pipe bend ranges from 20% for the single mitred pipe bend to 75% for the 3-weld mitred bend. Results also revealed that reducing the number of mitred welded joints, dominates reversed plasticity response at the expense of ratchetting response. Out-of-plane bending generally showed larger shakedown domain than the in-plane bending shakedown domain. Additionally, the shakedown domains for in-plane closing and opening moments are quite similar.

1. Introduction

Pressure vessel components are often subjected to the combined effect of simultaneous steady and cyclic load types. The combination of both the steady and the cyclic loads often results in exceeding the material initial yield strain (ε_0) within several parts or regions of the pressure vessel structure. It is the designer's objective to ensure that exceeding the initial yield strain (ε_0) would not lead to either development of progressive damage due to low cycle fatigue (reversed plasticity) and/or collapse due to incremental accumulation of plastic strain (ratchetting) associated with every load cycle. The upper ceiling of loads which does not cause either reversed plasticity and/or ratchetting responses is the elastic shakedown boundary. Pipe bends are typical examples of pressurized components. They are not only used to change direction of fluid flow, but to add necessary flexibility to the entire piping system. Since the first theoretical stress analysis of pipe bends published by von Kàrmàn in 1911, it has been well known that pipe bends acquire smaller flexural rigidity compared to straight pipes of the same material and dimensions. Therefore, pipe bends are considered amongst the critical pressurized components of a piping system. Mitred pipe bends are commonly installed within considerably large nominal pipe size diameter pipe networks typically found in power generation, chemical and pharmaceutical industries where manufacturing of 90-degree smooth pipe bends are difficult and/or uneconomic.

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Nomenclature

A_{IPO}	In-plane opening SD limit area
A_{OP}	Out-of-plane SD limit area
D	Outer pipe diameter
D_{m}	Straight pipe mean diameter
E	Modulus of elasticity
L	Length of straight pipe section
L_{i}	Length of pipe section
M	moment
M_{i}	Incremental moment
M_{y}	Yielding moment
P	Internal pressure
P_{y}	Yielding pressure
R	Pipe bend radius
R_{m}	straight pipe mean radius
S_{r}	Deviatoric residual stresses
a, b and c	Tensorial appendices
i	Elastic–plastic solution increment
t	Thickness
φ	Measured pipe section angle
α	Pipe bend angle
υ	Poisson's ratio
ε_0	Initial yield strain
σ_{E}	Elastic solution stress component
σ_{ELPL}	Elastic–plastic solution stress components
σ_{r_x}, σ_{r_y}, σ_{r_z}	Residual normal stress components
$\tau_{\mathrm{r}_{xy}}$, $\tau_{\mathrm{r}_{yz}}$, $\tau_{\mathrm{r}_{zx}}$	Residual shear stress components
σ_{r}	Residual stress component
σ_{y}	Yield strength

Abbreviations

FE	Finite element
SD	Shakedown
SMPB	Single mitred pipe bend
SPB	Smooth pipe bend
ST	Simplified technique
2-WMPB	2-Weld mitred pipe bend
3-WMPB	3-Weld mitred pipe bend
IPC	In-plane closing
IPO	In-plane opening
OP	Out-of-plane

2. Literature review

The term shakedown was initially introduced into the context of solid mechanics by Melan in 1936 through the shakedown lower bound theorem stated as follows: "For a given load set P, if any distribution of self-equilibrating residual stresses can be found (assuming perfect plasticity) which, when taken together with elastically calculated stresses, constitute a system of stresses within the yield limit, then P is a lower bound shakedown load set and the structure will shakedown". Additionally, the 2007 ASME Boiler and Pressure Vessel Code (2007) defines shakedown as follows: "the absence of significant progressive, cyclic, inelastic deformation."

Shakedown analysis of mitred pipe bends still represents a virgin territory for researchers as stated by Wood (2007). Most of the research performed to determine shakedown limit loads focused on pressure vessels (Leckie and Penny, 1967), nuclear reactor components (Bree, 1967), and aeronautical applications (Parkes and Benhamet al, 1964). Chang-Sik et al. (2008) generated shakedown boundaries for various 90-degree smooth elbows subjected to steady internal pressures and cyclic in-plane bending moments employing Abdalla et al.'s (2006) simplified technique. Chang-Sik et al. (2008) reported: "For more complex piping geometries such as pipe bends and nozzles, limited work on shakedown limit loads have been published in the literature such as Abdalla et al. (2007) and Carter (2005a, 2005b)".

Iterative elastic techniques were proposed to obtain rapid and approximate bounds for limit and shakedown loads utilizing the FE method. The Iterative elastic techniques begin with an initial elastic solution which is modified in an iterative manner, through a series of linear elastic finite element solutions, to redistribute stresses within the structure by changing the elastic modulii of all elements within the FE model. High-stressed elements have their modulii reduced while low-stressed elements have their modulii increased for the purpose of redistributing the stresses within the structure. The linear elastic FE iterations (solutions) proceed until a stress distribution in equilibrium with the externally applied load is reached. The iterative elastic techniques include the Elastic Compensation Method (ECM) introduced by Marriot (1988) and widely utilized by Mackenzie and Boyle (1993), the Dhalla reduction method proposed by Dhalla (1987), the Generalized LOcal Stress-Strain Redistribution Node "GLOSS R-Node" method proposed by Seshadri (1991), and the Linear Matching Method (LMM) proposed by Chen and Ponter (2001). The ECM is adopted by the ASME Boiler and Pressure Vessel Code (2007).

Abdalla et al. (2007) developed a direct non-cyclic numerical technique (called the simplified technique) to determine shakedown limit loads. The simplified technique was rigorously verified against classical shakedown benchmark problems (Abdalla et al., 2006, 2007, 2011a). Later Abdalla et al. (2006) extended the application of the simplified technique to a long radius 90-degree smooth pipe bend subjected to a spectrum of steady internal pressures and cyclic in-plane closing (IPC), in-plane opening (IPO) and out-of-plane (OP) bending moment loadings (Abdalla et al., 2011b) employing an elastic-perfectly-plastic material. Additionally, Abdalla et al. (2011b) performed a parametric study and generated Bree diagrams for 90-degree scheduled Nominal Pipe Size 10″ pipe bends namely: Schedule 20, Schedule 40 Standard, and Schedule 80 subjected to a spectrum of steady internal pressures and cyclic IPC, IPO, and OP bending moment loadings. Comparison of the generated Bree diagrams of the scheduled pipe bends revealed that as the wall thickness increased, both the limit loads and the shakedown limit loads increased as well.

Chen and Ponter (2001) published interesting experimental and FE simulation results on ratchetting of four low-carbon steel pressurized 90-degree pipe bend specimens subjected to cyclic reversed in-plane bending forces. A ratchetting boundary was predicted by Chen and Ponter (2001) through employing a modified form of the Ohno-Wang non-linear kinematic hardening rule. Abdalla et al. (2009) applied the simplified technique on one of four specimens where Chen and Ponter (2001) provided detailed results. The simplified technique showed very good correlation to Chen et al. (Boiler and Pressure Vessel Code, 2007) predicted ratchetting boundary for the medium to high steady internal pressure spectrum.

Recently, Abdalla et al. (2011c) determined the shakedown boundaries, limit loads, and elastic domains of a vessel–nozzle intersection subjected to a spectrum of steady internal pressures and cyclic in-plane bending moments applied on the nozzle. Additionally, the same vessel–nozzle intersection was later analysed