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# Nonlinear electromechanical fields and localized polarization switching of 1-3 piezoelectric/polymer composites

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ABSTRACT

This paper examines the nonlinear electromechanical response of 1-3 piezoelectric/polymer composites. The piezocomposites contain square or circular piezoelectric rods in an epoxy matrix. Experiments were conducted to measure the displacement versus electric field curves, using the device specimen of the 1-3 piezocomposites. Three dimensional finite element analysis was also carried out to study the electromechanical fields in the 1-3 piezocomposites by introducing a model for polarization switching. Comparison was then made between simulation and experiment.



## 1. Introduction

Piezoactive composites with 1-3 connectivity consist of piezoelectric rods or fibers in a polymer matrix. The advantages of the 1-3 piezocomposites are low acoustic impedance, low mechanical quality, and high or tailored electromechanical coupling coefficient. Therefore, they are well suited for ultrasonic transducers in nondestructive testing and medical imaging applications (Gebhardt et al., 2003). In recent years, a finite element model was developed to characterize the static response in 1-3 piezocomposites under electromechanical loading (Kar-Gupta and Venkatesh, 2007). Upon identifying 14 distinct fiber configurations, the influence of fiber distribution on the electromechanical response of model ceramic-matrix and polymer-matrix fiber composites was discussed. The modeling of 1-3 piezocomposites was also examined, and square arrangements of cylindrical piezoelectric fiber in the composites were considered (Berger et al., 2005). The effective elastic, piezoelectric, and dielectric constants were then calculated for different fiber volume fractions.

The aforementioned studies dealt with linear 1-3 piezocomposites. In some practical structures, one major concern has been the polarization switching of the piezoelectric rods or fibers. Also, theoretical and experimental investigations on the electromechanical response of the 1-3 piezocomposites are very limited.

In this paper, we investigate theoretically and experimentally the electromechanical response of 1-3 piezocomposites containing square or circular piezoelectric rods in an epoxy matrix. Many works used the linear constitutive relations for coupling response in the 1-3 piezocomposites, and hence the novelty of this work consists of the evolution of the nonlinear characteristics during polarization switching process and the comparison of the numerical values with the experimental data. Displacements were measured to characterize the response in the device specimen of the 1-3 piezocomposites. Nonlinear three dimensional finite element model incorporating the polarization switching mechanism was also used to predict the electromechanical fields in the 1-3 piezocomposites. Results produced by the model were then compared with experimental values.

## 2. Finite element analysis

### 2.1. Basic equations

Consider a piezoelectric material with no body force and free charge. The governing equations in the Cartesian coordinates $x_i (i = 1, 2, 3)$ are

$$\sigma_{ji,j} = 0 \tag{1}$$

$$D_{i,i} = 0 \tag{2}$$

where $\sigma_{ij}$ is the stress tensor, $D_i$ is the electric displacement vector, a comma denotes partial differentiation with respect to the coordinate $x_i$, and the Einstein summation convention over repeated indices is used. The relation between the strain tensor $\varepsilon_{ij}$ and the displacement vector $u_i$ is given by

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$$\varepsilon_{ij} = \frac{1}{2}(u_{j,i} + u_{i,j}) \tag{3}$$

and the electric field intensity vector is

$$E_i = -\phi_{,i} \tag{4}$$

where $\phi$ is the electric potential. In a ferroelectric material, domain switching leads to changes in the remanent strain $\varepsilon^{r}{}_{ij}$ and remanent polarization $P_i^r$. The constitutive relations can be written as

$$\sigma_{ij} = c_{ijkl}(\varepsilon_{kl} - \varepsilon_{kl}^{r}) - e_{kij}E_k \tag{5}$$

$$D_i = e_{ikl}(\varepsilon_{kl} - \varepsilon_{kl}^{r}) + \epsilon_{ik}E_k + P_i^r \tag{6}$$

In Eqs. (5) and (6), $c_{ijkl}$ and $e_{ikl}$ are the elastic and piezoelectric tensors, and $\epsilon_{ik}$ is the dielectric permittivity tensor. Valid symmetry conditions for the material constants are

$$c_{ijkl} = c_{jikl} = c_{ijlk} = c_{klij}, e_{kij} = e_{kji}, \epsilon_{ik} = \epsilon_{ki} \tag{7}$$

The constitutive Eqs. (5) and (6) for piezoelectric material poled in the $x_3$-direction are found in Appendix A.

The enhanced electromechanical field level results in localized polarization switching. When the electromechanical work exceeds a critical value, polarization switching occurs (Hwang et al., 1995). Thus the switching criterion is defined as

$$\sigma_{ij}\Delta\varepsilon_{ij} + E_i\Delta P_i \geq 2P^sE_c \tag{8}$$

where $P^s$ is a spontaneous polarization, $E_c$ is a coercive electric field, and $\Delta\varepsilon_{ij}$ and $\Delta P_i$ are the changes in the spontaneous strain and spontaneous polarization during switching, respectively. The values of $\Delta\varepsilon_{ij}$ and $\Delta P_i$ are given in Appendix B. The constitutive Eqs. (5) and (6) after polarization switching are given by

$$\sigma_{ij} = c_{ijkl}(\varepsilon_{kl} - \varepsilon_{kl}^{r}) - e'_{kij}E_k \tag{9}$$

$$D_i = e'_{ikl}(\varepsilon_{kl} - \varepsilon_{kl}^{r}) + \epsilon_{ik}E_k + P_i^r \tag{10}$$

The new piezoelectric constant $e'_{ikl}$ is given in Appendix C. The criterion of Eq. (8) is given on the basis of the total work done by electromechanical loads as the driving force for polarization switching. According to this criterion, the 180° switching and 90° switching occur when the mechanical work plus electrical work done reaches critical values corresponding to 180° and 90° switchings, respectively. The model is simple and quite successful at predicting the homogeneous average response of PZT material systems (Steinkopff, 1999; Hayashi et al., 2003), so we choose the switching criterion defined in Eq. (8).

### 2.2. Computational model

The ANSYS finite element analysis software was used in this investigation. A three-dimensional finite element model was necessary to accurately represent the electromechanical field distributions of the 1-3 piezocomposites. Consider two classes of 1-3 piezocomposites as shown in Fig. 1. A rectangular Cartesian coordinate system ($x$, $y$, $z$) is used with the $z$-axis coinciding with the poling direction. $4N^2$ square piezoelectric rods of side $a$ and height $h$ (Fig. 1(a)) or circular piezoelectric rods of diameter $d$ and height $h$ (Fig. 1(b)) are embedded in a polymer matrix, and arranged in a square array; where $N$ is half of the number of rods aligned in the $x$- or $y$-direction. So, the height of the 1-3 piezocomposites is $h$. Repeating units of the composites with $4N^2$ square and circular piezoelectric rods occupy the region ($0 \leq x \leq a + a_g$, $0 \leq y \leq a + a_g$, $0 \leq z \leq h/2$) and ($0 \leq x \leq d + d_g$, $0 \leq y \leq d + d_g$, $0 \leq z \leq h/2$), and the volume fractions $V_f$ of the square- and circular-type 1-3 piezocomposites are $a^2/(a + a_g)^2$ and $\pi d^2/4(d + d_g)^2$, respectively, where $a_g$ and $d_g$ are the gaps between the piezoelectric rods.

Electrodes lie in the top and bottom ends of each piezoelectric rod. The electric potential on the top electrode surfaces equals the applied voltage, $\phi = V_0$. The bottom electrode surfaces are connected to the ground, so that $\phi = 0$. Each element was defined by eight-node 3-D coupled field solid for the piezoelectric rods and eight-node 3-D structural solid for the polymer matrix. Because of the geometric and loading symmetry, only one-eighth of the specimen needs to be analyzed. The mechanical boundary conditions include the traction-free condition on the top surface at $z = h/2$ and the zero-displacement conditions on the $x = 0$, $y = 0$ and $z = 0$ faces. Also included are the traction-free conditions on the side surfaces at $x = N(a + a_g)$, $y = N(a + a_g)$ for the square-type 1-3 piezocomposite or at $x = N(d + d_g)$, $y = N(d + d_g)$ for the

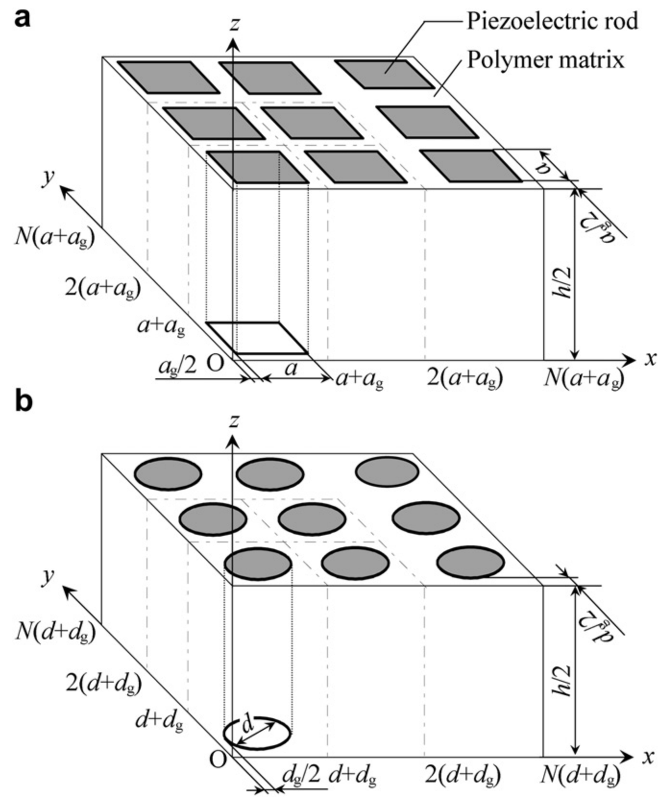


**Fig. 1.** Illustration of 1-3 piezocomposite with (a) square and (b) circular rods.

**Table 1**
Material properties of C-6.

| | Elastic stiffnesses ($\times 10^{10}$ N/m$^2$) | | | | | Piezoelectric coefficients (C/m$^2$) | | | Dielectric constants ($\times 10^{-10}$C/Vm) | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $c_{11}$ | $c_{33}$ | $c_{44}$ | $c_{12}$ | $c_{13}$ | $e_{31}$ | $e_{33}$ | $e_{15}$ | $\epsilon_{11}$ | $\epsilon_{33}$ |
| C-6 | 12.3 | 11.2 | 1.9 | 7.7 | 8.0 | −7.3 | 17.2 | 14.5 | 92.0 | 66.3 |