European Journal of Mechanics A/Solids 29 (2010) 420–439

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

Image singularities of planar magnetoelectroelastic bimaterials for generalized line forces and edge dislocations

Jui-Mu Lee, Chien-Ching Ma*

Department of Mechanical Engineering, National Taiwan University, Taipei 10617, Taiwan, ROC

A R T I C L E I N F O

Article history:
Received 11 November 2008
Accepted 23 October 2009
Available online 30 October 2009

Keywords:
Magnetoelectroelastic bimaterials
Edge dislocation
Image force
Image singularities

A B S T R A C T

This study presents two-dimensional explicit full-field solutions of transversely isotropic magnetoelectroelastic bimaterials subjected to generalized line forces and edge dislocations using the Fourier-transform technique. One of the major objectives of this study is to analyze the physical meaning and the structure of the solution. Complete solutions for this problem consist only of the simplest solutions for an infinite medium. The solutions include Green's function of originally applied singularities in an infinite medium and thirty-two image singularities which are induced to satisfy interface continuity conditions. It is shown that the physical meaning of the solution is the image method. The mathematical method used in this study provides an automatic determination for the locations and magnitudes of image singularities. The locations and magnitudes of image singularities are dependent on the roots of the characteristic equation for bimaterials. The number and distribution for image singularities are discussed according to characteristic roots features. With the aid of the generalized Peach–Koehler formula, the explicit expressions of image forces acting on generalized edge dislocations are easily derived from the full-field solutions of the generalized stresses. Numerical results for the full-field distributions of stresses, electric fields, and magnetic fields in bimaterials are presented. The image forces and equilibrium positions of one dislocation, two dislocations, and an array of dislocations are presented by numerical calculations and are discussed in detail.

1. Introduction

Smart or intelligent materials, such as magnetoelectroelastic solids exhibiting full coupling between mechanical, electric, and magnetic fields, have been drawn considerable discussions in the past decade. Since these materials simultaneously possess piezoelectric, piezomagnetic, and high magnetoelectric coupling effects, they can convert energy from one form to the other (among magnetic, electric, and mechanical energies). Due to the bidirectional nature of this energy exchange, magnetoelectroelastic materials are very sensitive to elastic, electric, and magnetic fields in the environment. Consequently, they are extensively used as magnetic field probes, electric packaging, acoustic, hydrophones, medical ultrasonic imaging, sensors, and actuators with the magnetoelectromechanical energy conversion.

For a two-dimensional planar problem of an infinite plane, the deformations, electric fields, and magnetic fields in magnetoelectroelastic solids produced by generalized line forces or dislocations are well-known. This work shows that the solution of planar magnetoelectroelastic bimaterials obtained from the mathematical method proposed in this study has its own physical meaning, the image method. The image method is a technique that uses a simple fundamental solution in an infinite plane to construct the solution for other more complicated problems. On the other hand, some defects (such as dislocations and cracks) are induced during their manufacturing processes or during service by the mechanical, electrical, or magnetic loadings. In other words, dislocations are material defects that can adversely influence electronic device performance. Therefore, a detailed understanding of magnetoelectroelastic material behavior subjected to dislocations is needed. Investigating dislocation movement due to applied loadings and other defects is also important. Solutions for the generalized stresses induced by a dislocation can be used to provide a direct means of determining the generalized Peach–Koehler (or image) force, which is of direct relevance in understanding real dislocation behavior characteristics.

The two-dimensional in-plane and anti-plane problems for anisotropic elastic solids can be solved by a superposition of simple image singularities over the plane (Ting, 1996). The solution for a line dislocation in an infinite anisotropic medium has been obtained by Eshelby et al. (1953), Stroh (1958), and Willis (1970). Green's function for two-dimensional deformations of an

* Corresponding author. Tel.: +886 2 23659996; fax: +886 2 23631755.
E-mail address: ccma@ntu.edu.tw (C.-C. Ma).

0997-7538/$ – see front matter © 2009 Elsevier Masson SAS. All rights reserved.
doi:10.1016/j.euromechsol.2009.10.012

anisotropic elastic half-space subjected to a line force and/or a line dislocation inside the half-space has been considered by Willis (1970), Barnett and Lothe (1974), and Ting and Barnett (1993). The image singularities of Green's functions for isotropic elastic half-plane and bimaterials, which are the degenerate case of anisotropic materials, were discussed in detail by Ma and Lin (2001, 2002). Ma and Lu (2006) recently presented theoretical analysis of screw dislocations and image forces in the anisotropic multilayered medium. For piezoelectric materials with electromechanical coupling, Green's functions have also been investigated by many authors. Pak (1990) considered a screw dislocation in an unbounded piezoelectric medium and derived the generalized Peach–Koehler forces acting on a screw dislocation subjected to external loads. The solution obtained was then used to derive the image fields for a half-space with traction-free boundary. Based on the extended Stroh formulism, Green's functions were presented for the problem of defects interacting with a line dislocation in two-dimensional infinite anisotropic piezoelectric medium (Huang and Kuang, 2001; Chen et al., 2004). The interaction of defects and screw dislocations under anti-plane mechanical and in-plane electrical loading in piezoelectric bimaterials was studied by Wu et al. (2003), Liu and Wang (2004), and Wang and Sudak (2007). The expressions for the image force acting on the screw dislocation due to its interaction were also derived.

The development of piezoelectric–piezomagnetic composites has its roots in the early work of van Suchtelen (1972) who reported that the piezoelectric–piezomagnetic composites exhibit a different material property, i.e. the magnetoelectric coupling effect. Furthermore, composites made of piezoelectric/piezomagnetic materials exhibit magnetoelectric coupling effect that is not present in single-phase piezoelectric or piezomagnetic materials. In the past decade, much attention has been paid to predict the effective properties of magnetoelectroelastic composites according to the theories of micromechanics. Nan (1994) proposed micromechanics models to determine the effective properties of piezoelectric/piezomagnetic composites. For fibrous composites, exact connections among effective magnetoelectroelastic modules have been derived by Benveniste (1995) using a uniform field concept. Huang and Kuo (1997), Huang et al. (1998) established a unified and analytical method for determining the magnetoelectroelastic fields for an ellipsoidal inclusion and the effective properties of a piezoelectric (inclusion)–piezomagnetic (matrix) composite.

For the study of dislocation and image forces in magnetoelectroelastic solids, relatively little work has been done. Guan and He (2005) obtained expressions of elastic displacement, stress, electric displacement, electric potential, magnetic induction and magnetic potential for the two-dimensional plane problem of a transversely isotropic magnetoelectroelastic half-plane medium subjected to a point force along the x-axis. Hao and Liu (2006) investigated the interaction between a screw dislocation and a semi-infinite interfacial crack in transversely isotropic magnetoelectroelastic bimaterials. Green's function for two-dimensional magnetoelectroelastic full-, half-, and bimaterials-planes containing cracks or inclusions with a line force and/or a line dislocation has been considered by Liu et al. (2001), Jiang and Pan (2004), and Tian and Gabbert (2005). Analytical full-field solutions of a magnetoelectroelastic layered half-plane subjected to generalized concentrated forces and screw dislocations were presented by Lee and Ma (2007). The solution obtained was then used to derive image forces of screw dislocations in a layered half-plane by Ma and Lee (2007). The corresponding in-plane problem of a magnetoelectroelastic layered half-plane subjected to generalized loadings and edge dislocations was recently presented by Ma and Lee (2009).

The problem of transversely isotropic magnetoelectroelastic bimaterials subjected to a generalized line force and edge dislocation is analyzed in the present paper. This work uses the Fourier-transform method to develop an effective analytical methodology to construct full-field explicit solutions for this problem. Analytical solutions for the generalized displacements and stresses obtained in this study are expressed in an explicit closed-form. The complete solutions consist only of the simplest solutions for an infinite homogeneous medium with generalized line loadings. The physical meanings of the solution can be regarded as the image method for the planar magnetoelectroelastic problem. Based on the full-field solutions of the generalized stresses and the generalized Peach–Koehler equation, the explicit expressions of image forces exerted on generalized edge dislocations (one dislocation, two dislocations, and an array of dislocations) are derived and are easily used for numerical calculation. For numerical examples, the full-field distributions of stresses, electric fields, and magnetic fields in bimaterials subjected to line forces or edge dislocations are presented. We focus our attention also on the numerical calculation of image forces for one edge dislocation, two edge dislocations, and an array of edge dislocations. The equilibrium position and the stability of edge dislocations embedded in magnetoelectroelastic bimaterials are investigated and interesting phenomena of image forces are presented.

The method of image is a technique that uses the superposition of known solution to construct the solution to other complicated problems. One of the main contributions of this paper is to provide detailed information of the locations and magnitudes of image singularities. It is noted that the locations of image singularities are dependent on the characteristic roots of bimaterials. According to the characteristic roots, the number and distributed locations of image singularities are discussed in detail. In general, there are thirty-two image singularities which will be induced in the biomaterial. The explicit locations of all the image singularities are presented in this study which are fundamentally important to understand the basic feature of the problem. Furthermore, it is also proved in this study that the resultant forces and dislocations for all the image singularities of the bimaterial are identical to the applied generalized forces and dislocations. This study provides a complete discussion on the structure of the solution for magnetoelectroelastic biomaterial.

2. Basic equations and general solutions

2.1. Governing equations

A Cartesian coordinate system is used in the analysis. Let σ_{ij}, ε_{ij}, D_i, E_i, B_i, and H_i denote the stress tensor, the strain tensor, electrical displacements, electric fields, magnetic inductions, and magnetic fields, respectively. For an anisotropic, linearly magnetoelectroelastic solid, the coupled constitutive relations at a constant temperature are

$$\varepsilon_{ij} = s_{ijkl}\sigma_{kl} + g_{kij}D_k + h_{kij}B_k, \tag{2.1a}$$

$$(-E_i) = g_{ikl}\sigma_{kl} - \alpha_{ik}D_k - \gamma_{ik}B_k, \tag{2.1b}$$

$$(-H_i) = h_{ikl}\sigma_{kl} - \gamma_{ik}D_k - \beta_{ik}B_k, \tag{2.1c}$$

where s_{ijkl}, α_{ik}, and β_{ik} are the elastic compliance, dielectric impermeability, and reluctivity, respectively; g_{kij}, h_{kij}, and γ_{ik} are the piezoelectric strain constants, piezomagnetic strain constants, and the magnetoelectric constants, respectively. Constitutive equations use $-E_i$ and $-H_i$ instead of E_i and H_i, because they enable the construction of a symmetric matrix of constitutive modules. We will focus on a transversely isotropic magnetoelectroelastic material for the poling direction along the z-direction. In such a case, constitutive equations (2.1a)–(2.1c) can be represented as the following matrix formulation: