European Journal of Mechanics A/Solids 28 (2009) 1079–1087

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

# Shearing effects on the breathing mechanism of a cracked beam section in bi-axial flexure

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ARTICLE INFO

*Article history:*
Received 21 April 2008
Accepted 10 May 2009
Available online 27 May 2009

*Keywords:*
Beam finite element
Breathing crack
Unilateral contact
Shearing effects
Convex analysis

ABSTRACT

The main purpose of this paper is to complete the works presented by Andrieux and Varé (2002) and El Arem et al. (2003) by taking into account the effects of shearing in the constitutive equations of a beam cracked section in bi-axial flexure. The paper describes the derivation of a lumped cracked beam model from the three-dimensional formulation of the general problem of elasticity with unilateral contact conditions on the crack lips. Properties of the potential energy and convex analysis are used to reduce the three-dimensional computations needed for the model identification, and to derive the final form of the elastic energy that determines the nonlinear constitutive equations of the cracked transverse section. We aim to establish a relation of behavior between the applied forces and the resulting displacements field vectors, which is compatible with the beams theory in order to allow the model exploitation for shafts dynamics analysis. The approach has been applied to the case of a cracked beam with a single crack covering the half of its circular cross section.



## 1. Introduction: state of the art

Since the early 1970s when investigations on the vibrational behavior of cracked rotors began, numerous papers on this subject have been published, as a literature survey by Dimarogonas (1996) shows. The analysis of the behavior of cracked rotating machinery shafts is a complex structural problem. It requires, for a relevant description, a fine and precise modeling of the shaft and cracks in order to allow the identification and calculation of the parameters characterizing their presence.

Researchers dealing with the problem of rotating cracked beams recognize its two main features, namely:

- the determination of the local flexibility of the beam cracked section;
- the consideration of the opening – closing phenomenon of the crack during the shaft rotation, commonly called breathing mechanism of the crack, and responsible of the system nonlinear behavior: when the shaft is rotating, then the crack opens and closes according to the stresses developed in the cracked surface. If these stresses are extensive, then the crack opens, resulting in a reduced shaft stiffness. When the stresses are compressive, then the crack remains closed and the shaft has the same stiffness as the non-cracked shaft. Thus, the system stiffness is depending on the cracked section position (Fig. 1).

This breathing mechanism depends on the shaft rotation in the case when the static deflection dominates the vibration of the rotating shaft. This is a very common situation in large turbine-generator rotors.

During the last three decades, great attention has been paid by several research scientists to the analysis and diagnosis of cracks in rotating machinery. The excellent review papers by Entwistle and Stone (1990), Dimarogonas (1996), Wauer (1990a) and Gasch (1993) cover many aspects of this area and summarize the most relevant analytical, experimental and numerical works conducted in the last three decades and related to the cracked structures modeling.

There have been different attempts to quantify local effect introduced by cracks. The analysis of the local flexibility of a cracked region of structural element was quantified in the 1950s by Irwin (1957a,b), Bueckner (1958), Westmann and Yang (1967) by relating it to the Stress Intensity Factors (SIF). Afterwards, the efforts to calculate the SIF for different cracked structures with simple geometry and loading was duplicated (Tada et al., 1973; Bui, 1978).

Most researchers agree with the application of the linear fracture mechanics theory to evaluate the local flexibility introduced by

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doi:10.1016/j.euromechsol.2009.05.004

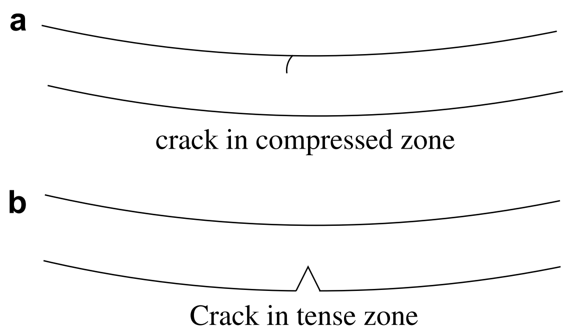


**Fig. 1.** Crack breathing mechanism.

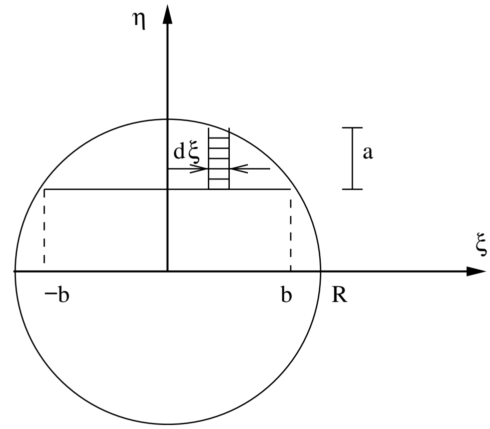


**Fig. 2.** Geometry of the shaft cracked section.

the crack (Gross and Srawley, 1965; Anifantis and Dimarogonas, 1983; Dimarogonas and Paipetis, 1983; Dimarogonas, 1996; Papadopoulos and Dimarogonas, 1987a,b,c; Papadopoulos, 2004). Obviously, the first work was done in the early 1970s by Dimarogonas (1970, 1971) and Pafelias (1974) at the General Electric Company. The energy release rate approach combines the linear fracture mechanics to rotordynamics theory in order to calculate the compliance caused by a transverse surface crack affecting a rotating shaft. A good review on this method is presented by Papadopoulos (2008).

For an elastic structure, the additional displacement $\boldsymbol{u}$ due to the presence of a straight crack of depth $a$ under the generalized loading $\boldsymbol{P}$ is given by the Castigliano theorem

$$u = \frac{\partial}{\partial \boldsymbol{P}} \int_0^a \boldsymbol{G}(a)\mathrm{d}a \tag{1}$$

$\boldsymbol{G}$ is the energy release rate related to the SIF by the Irwin formula (Irwin, 1957a). Then, the local flexibility matrix coefficients are obtained by

$$c_{ij} = \frac{\partial^2}{\partial P_i \partial P_j} \int_0^a \boldsymbol{G}(a)\,\mathrm{d}a, \quad 1 \le i,j \le 6 \tag{2}$$

Extra diagonal terms of this matrix are responsible for longitudinal and lateral vibrations coupling that could be with great interest when dealing with cracks detection.

In two technical notes of the NASA, Gross and Srawley (1964, 1965) computed the local flexibility corresponding to tension and bending including their coupling terms. This coupling effect was observed by Rice and Levy (1972) in their study of cracked elastic plates for stress analysis.

Dimarogonas and his co-workers (Dimarogonas, 1982; Dimarogonas and Paipetis, 1983; Dimarogonas, 1987, 1988), and Anifantis and Dimarogonas (1983) introduced the full $(6 \times 6)$ flexibility matrix of a cracked section. They noted the presence of extra diagonal terms which indicate the coupling between the longitudinal and lateral vibrations. Papadopoulos and Dimarogonas (1987a,b,c), and Ostachowicz and Krawwczuk (1992) computed all the $(6 \times 6)$ flexibility matrix of a Timoshenko beam cracked section for any loading case.

However, there are no results for the SIF for cracks on a cylindrical shaft. Thus, Dimarogonas and Paipetis (1983) have developed a procedure which is commonly used in FEM software: the shaft was considered as an assembly of elementary rectangular strips where approximation of the SIF using fracture mechanics results remains possible (Fig. 2). The SIF are obtained by integration of the energy release rate on the crack tip by

$$c_{ij} = \frac{\partial^2}{\partial P_i \partial P_j} \int_0^a \int_{-b}^b \boldsymbol{G}(a,\xi)\,\mathrm{d}\xi\,\mathrm{d}a \tag{3}$$

Although it offers the advantage of being easy to insert in a numerical algorithm, this method has some limitations. In fact, some numerical problems were reported by Abraham et al. (1994) when the depth of the crack $a$ exceeds the section radius $R$. Dimarogonas (1994), in his reply, stated that this divergence does not reflect reality; it is due to the assumption of a 2D stress distribution which is not valid near the ends of the crack tip.

Papadopoulos (2004) suggested to consider Equation (3) with:

$$0.90b \le |\xi| \le 0.95b$$

when the crack depth $a$ exceeds the shaft radius $R$.

Wauer (1990b) explored the dynamics of a cracked, distributed parameter rotor component. The proposed model is a rotating Timoshenko shaft which is also flexible in extension and torsion. The stiffness matrix is constructed using the energy release rate approach as described in papers by Papadopoulos and Dimarogonas (1987a,b), and Gudmundson (1983). The geometric discontinuity due to the crack is replaced by a load discontinuity. The procedure reduces the problem to equations for one uniform beam with a modified load distribution. The proposed approach is a powerful instrument to obtain approximate results by a relatively small calculation expense. Although the shearing is considered in this approach, the author did not discuss the importance of its effects on the crack breathing mechanism.

## 2. Review of the Andrieux and Varé model

An original method for deriving a lumped model for a cracked beam section was proposed by Andrieux and Varé (2002). Based on three-dimensional computations, the procedure incorporates more realistic behavior of the cracks than the previous models, namely the unilateral contact conditions on the crack lips and their breathing mechanism under variable loading. The method was derived from three-dimensional formulation of the general problem of elasticity with unilateral contact conditions on the crack lips. The authors established properties of the potential energy of this problem to reduce the amount of computation required for its determination in the case of a beam containing cracks of any shape and number. Convex analysis was also used to derive the final form of the energy that determines the nonlinear constitutive equations of the section of the beam which was incorporated in a FE analysis code. Great attention is paid to the capability of such a model to