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Crack tip fields in soft elastic solids subjected to large quasi-static deformation — A review

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ABSTRACT

We review analytical solutions for the asymptotic deformation and stress fields near the tip of a crack in soft elastic solids. These solutions are based on finite strain elastostatics and hyperelastic material models, and exhibit significantly different characteristics than the classical crack tip field solutions in linear elastic fracture mechanics. Specifically, we summarize some available finite strain crack tip solutions for two dimensional cracks, namely that plane strain, plane stress, and anti-plane shear cracks in a certain class of homogeneous materials. Interface cracks between soft elastic solids and a rigid substrate are also discussed. We focus on incompressible material models with various degrees of strain stiffening effect such as generalized neo-Hookean model, exponentially hardening model and Gent model. We also explored the physical implications of the crack tip fields, and highlighted pitfalls in the applications of these solutions, particularly the J-integral and the distribution of true stress in the deformed configuration which have not been discussed in the literature.

1. Introduction

Studies on the fracture of soft materials can be dated back to the 1950s where the first attempt to quantitatively characterize and understand the rupture behaviors of rubber was made in a number of seminal works [1–5]. Rivlin and Thomas [1] conducted fracture experiments on natural rubber and found that the critical fracture load for various testing configurations and sample geometries was dictated by a single characteristic tearing energy, or fracture energy. Based on this finding, they proposed an energy based fracture criterion where the onset of growth of a pre-existing crack occurs when the applied energy release rate is equal to a characteristic energy/area which in modern terms is called fracture energy — a characteristic of the material. Numerous experiments were carried out to measure the fracture energy of rubbers and the results indicated strong rate and temperature dependency for the fracture energy [6], which are commonly attributed to energy dissipation due to viscoelasticity. Recently, interest in soft material fracture has been renewed by the development of hydrogels [7–11] or polymers [12] with high extensibility and fracture toughness. Examples include the double-network gel [7], the hybrid polyacrylamide–alginate gel [8] and the ionically crosslinked triblock copolymer gel [10]. These soft and yet tough materials, when combined with additional functionalities, have great potentials in a wide range of applications such as bio-adhesives [13], biomedical implants [14], tissue engineering [15], and soft actuators [16].

The most common way to test the fracture behaviors of soft materials is to first introduce a sharp crack in a specimen and then mechanically load the specimen until

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List of main symbols and abbreviations

$\eta_1(\theta, n)$	Dimensionless function in (42b)
$\eta_2(\theta, n)$	Dimensionless function in (42c)
κ	$\kappa = 1 - 1/n$ in (36b).
μ	Small strain shear modulus
(ρ, φ)	Polar coordinates at deformed crack tip (see Fig. 1(B))
$\sigma_{ij}(\sigma_{\alpha\beta})$	Nominal stress or the first Piola–Kirchhoff stress
$\tau_{ij}(\tau_{\alpha\beta})$	True stress or Cauchy stress
$\omega(\theta, n)$	Dimensionless function in (36c)
ξ_0	Intermediate variable defined in (37c)
A, B	Coefficients of strain energy density in plane strain (see (34))
A_2	Coefficient in plane stress crack tip solution for exponential solids (see (69a))
a	Coefficient in crack tip solution (see (35b), (58b) and (76))
b_0	Coefficient in plane strain crack tip solution (see (35a))
b	Material parameter in the generalized neo-Hookean model (see (13))
c	Coefficient in plane stress crack tip solution (see (58a))
d	Exponent of the crack tip solution of y_1 in plane stress (see (58a))
$F_{\alpha\beta}$	2-D deformation gradient
EM	Exponentially Hardening Model introduced in (14)
FEM	Finite Element Method
GM	Gent Model introduced in (15)
GNM	Generalized neo-Hookean Model introduced in (13)
$g(\theta, n)$	Dimensionless function describing the angular variation of y_1 in plane stress (see (58a))
$H(\theta, n)$	Dimensionless function introduced in (37a)
$\hat{I}_n(\theta)$	Dimensionless function introduced in (42a)
$I_1 = I$	First invariant of the right Cauchy–Green tensor C_{ij} (see (6))
I_2	Second invariant of the right Cauchy–Green tensor (see (6))
J	J-integral defined in (96a) and (96b)
J_n	Material parameter in the exponential model or Gent solid (see (14) and (15))
K_I, K_{II}, K_{III}	Mode I, II and III stress intensity factors in linear elastic fracture mechanics
LEFM	Linear Elastic Fracture Mechanics
m	$m = 1 - 1/(2n)$ in (37b)
n	Material parameter describing the degree of strain hardening (see (13) and (34))
n^*	Critical strain hardening exponent at which the plane stress crack tip field changes behavior (see (58a) and $n^* \approx 1.46$).
p	Pressure required to enforce incompressibility
$P(\theta, n)$	Angular variation of the pressure fields for $1/2 < n < 3/2$ in (44a).
(r, θ)	Polar coordinates at undeformed crack tip (see Fig. 1(A))
$U(\theta, n)$	Dimensionless function introduced in (36a)
u_i	Components of the displacement of a material point
W	Strain energy density
X_α	Material coordinates (see Fig. 1(A))
x_α	Deformed coordinates with origin at undeformed crack tip
y_α	Deformed coordinates with origin at deformed crack tip (see Fig. 1(B))
y_α^*	Canonical form of mixed-mode crack tip solutions (see (55))

the crack propagates forward. Compared to other fracture tests which relies on the presence of initial defects to induce local failure mechanisms such as cavitation [17–19], the crack geometry provides a well-defined initial configuration where the stress distribution near the crack tip is similar, independent of geometry. As long as the deviation from the continuum description occurs over a region small in comparison with typical specimen dimensions, the amplitude of the near tip stress fields controls the local failure processes such as breaking of polymer chains, and growth and coalescence of cavities. However, unlike stiff materials such as metal and ceramics, cracks in soft and tough materials can become highly deformed and blunted before propagation [20]. This can be further complicated by time-dependent effects such as viscoelasticity [6] and poroelasticity [21,22]. Therefore, the classical solutions for crack tip fields in linear elastic fracture mechanics (LEFM), or the K-field [23], which were based on the assumption of infinitesimal deformation, are not sufficient to describe the local crack tip stress field in soft materials. For example, experimental observations and theoretical analyses suggested that the LEFM crack tip fields broke down for dynamic [24–28] or quasi-static cracks [25,28,29] in soft polymeric gels.

A first step towards understanding crack tip fields in soft materials is to take large deformation effects into consideration by adopting the framework of finite strain elastostatics. The earliest effort along this direction was due to Wong and Shield [30], where closed form approximate solutions for a plane stress crack in a sheet of incompressible neo-Hookean material were presented. After that, Knowles and Sternberg [31,32] performed the first systematic asymptotic analysis of near-tip fields for a plane strain crack under symmetric opening deformation (Mode-I). They considered a class of power-law stiffening compressible hyperelastic solids, and show that the severity of the singular stress and deformation fields depends on material stiffening. The amplitudes of these singular stress and deformation fields are governed by far-field loading conditions and are analogous to the stress intensity factors in LEFM [33,23]. These pioneering works inspired further studies to understand the local stress and