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Multiple crack problems in nonhomogeneous orthotropic planes under mixed mode loading conditions

M.M. Monfared, M. Ayatollahi *

Faculty of Engineering, University of Zanjan, P.O. Box 45195-313, Zanjan, Iran

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ABSTRACT

Analytical method based on continuous dislocation technique is developed to examine the mixed mode behavior of a functionally graded orthotropic plane. First, the elastic stress fields in a functionally graded orthotropic infinite plane containing single Volterra types climb and glide edge dislocations are obtained. Using the dislocation solutions, the problem of interacting cracks is reduced to a system of singular integral equations for dislocation density functions on the surfaces of smooth cracks which are solved numerically for the dislocation density. Then, the dislocation density function is applied to calculate the stress intensity factors for several different cases of crack configurations and arrangements.

1. Introduction

The advent of materials with continuously varying volume fractions, the so-called functionally graded materials (FGMs), and their technological potential, introduced and applied in aerospace, energy, electronics, and other applications, stimulated a fair amount of research in this area. The stress analysis of the functionally graded orthotropic materials with multiple cracks under in-plane loading is of considerable importance in the design of safe structures. This is mainly due to the ever increasing usage of non-homogenous orthotropic materials in the modern technology. Because of the nature of such techniques used in processing, the graded materials are not often isotropic. For example, functionally graded materials processed by using a plasma spray technique have a generally lamella structure. Such materials are not isotropic, but orthotropic, with material directions that can be considered perpendicular to one another in an initial approximation. This implies that the orthotropic properties should be considered in studying the mechanics of FGMs.

Sih et al. [1] investigated the mixed mode problem for a circular arc crack in a sheet under biaxial traction. The strength of stress singularities at crack tips in-plane problem and plate bending problem has been studied. The stress analysis in a nonhomogeneous elastic plane with cracks was discussed by Erdogan [2]. It was shown that the stress singularity is in the form of $r^{-1/2}$, r being the distance from the crack tip. Experimental stress intensity factors for two interacting straight cracks in a homogeneous plane have been determined by Soozani et al. [3]. The plane elasticity problem for a nonhomogeneous medium containing a crack was re-examined by Delale and Erdogan [4]. In this work the integral equation for a crack problem was obtained. The results were shown that the effect of the Poisson's ratio and consequently that of the thickness constraint on the stress intensity factors are rather negligible and the results are highly affected by the FG parameter. Delale and Erdogan [5] reconsidered the mixed mode problem for an interface crack in a nonhomogeneous elastic medium. The problem was solved for various values of the nonhomogeneous parameter. Further results for a crack in a nonhomogeneous material were

* Corresponding author. Tel.: +98 24 3305 2488; fax: +98 24 3305 3204.
E-mail address: mo.ayatollahy@gmail.com (M. Ayatollahi).

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Nomenclature

a half-lengths of straight cracks and radius of circular arc cracks
$A_1(s),\ A_2(s),\ A_3(s),\ A_4(s)$ unknowns coefficients
$b_x,\ b_y$ dislocation densities
$B_n,\ B_s$ dislocation densities in orthogonal coordinates $s,\ n$
$C_{11}(y),\ C_{22}(y),\ C_{12}(y),\ C_{66}(y)$ elastic moduli
$g_s(t),\ g_n(t)$ regular terms of dislocation densities in orthogonal coordinates $s,\ n$
$H(x)$ Heaviside step function
$K_{IL},\ K_{IIL},\ K_{IR},\ K_{IIR}$ stress intensity factors of left and right side of crack
K_0 stress intensity factor of a crack in infinite plane
$K_{11ik}(s,t),\ K_{12ik}(s,t),\ K_{21ik}(s,t),\ K_{22ik}(s,t)$ kernels of integral equations
N number of cracks
$u,\ v$ displacement components
$u_{si},\ u_{ni}$ displacement components in orthogonal coordinates $s,\ n$
$x,\ y$ coordinates
$\alpha(t),\ \beta(t)$ parametric equations of the crack
$\delta(s)$ Dirac delta function
$\varepsilon_{xx},\ \varepsilon_{yy},\ \gamma_{xy}$ strain components
θ crack orientation
κ Kolosov constant
$\sigma_n,\ \sigma_s$ stress components in orthogonal coordinates $s,\ n$
$\sigma_{xx},\ \sigma_{yy},\ \sigma_{xy}$ stress components
σ_0 applied tractions at infinity
ϕ Airy stress function

given by Erdogan et al. [6]. In this study, the plane elasticity problem for two bonded half-planes containing a crack perpendicular to the interface was considered. Ozturk and Erdogan [7] studied the axisymmetric crack problem in a nonhomogeneous medium. The main results were the stress intensity factors as a function of the nonhomogeneous parameter for various loading conditions. Konda and Erdogan [8] considered the mixed mode plane strain problem for an arbitrarily orientated crack in a nonhomogeneous medium. Erdogan [9] examined the influence of the nonhomogeneous material on the asymptotic stress state near the crack tips. Chen and Erdogan [10] investigated the problem of the interface crack in a nonhomogeneous coating bonded to a homogeneous substrate. The symmetric mode I crack problem in an inhomogeneous orthotropic medium, was examined by Ozturk and Erdogan [11]. It has been shown that in the mode I crack problem for an orthotropic inhomogeneous medium, the Poisson's ratio has only a negligible influence on the stress intensity factors but the effect of the material inhomogeneity was quite significant. The in-plane crack problem for an inhomogeneous orthotropic medium was treated by Ozturk and Erdogan [12]. It was found that the stress intensity factors are increased with increasing materials inhomogeneous parameter and with decreasing stiffness ratio. Huang and Kardomateas [13] obtained modes I and II stress intensity factors in a fully anisotropic strip with a central crack. The effects of the material anisotropy on the mixed mode stress intensity factors of a center crack have been studied in the literature. Dag and Erdogan [14] studied the problem of a surface crack in a semi-infinite graded under general loading conditions. Kim and Paulino [15] conducted finite element studies to evaluate the stress intensity factor of the orthotropic functionally graded material. They investigated the effects of boundary conditions, crack-tip mesh discretization and materials properties on the fracture behavior of the medium. Rao and Rahman [16] solved crack problems for isotropic functionally graded materials under mixed mode loading conditions where they used Galerkin-based meshless method for calculating the stress intensity factors. The plane elasticity problem of an arbitrarily orientated crack in an FGM layer bonded to homogeneous half-plane was considered by Long and Delale [17]. It has been shown that the crack length, orientation and the nonhomogeneity parameter of the layer have a significant effect on the fracture of the FGM layer. Ma et al. [18] analyzed the mixed mode crack problem for a functionally graded orthotropic medium under time-harmonic loading. In this study, the effects of the material properties and the crack configuration on the stress intensity factors were investigated. A numerical procedure based on the concept of the J-integral, for computation of the mixed mode stress intensity factors for curved cracks was obtained by Chang an Wu [19]. Dag et al. [20] analyzed the mixed mode fracture problem of orthotropic functionally graded material under mechanical and thermal loading conditions. Modes I and II stress intensity factors for cracks and also the hoop stress for cavities in an orthotropic plane were obtained by Fotuhi and Fariborz [21]. Faal and Fariborz [22] employed the distributed dislocation technique to analyze an orthotropic plane having multiple cracks. The results were used to evaluate modes I and II stress intensity factors. Hongmin et al. [23] adopted the Winner-Hopf technique to analyze the problem of semi-infinite cracks in an infinite functionally graded orthotropic material. Steady state interaction between multiple cracks in an infinite plane under in-plane time-harmonic loads was studied by Ayatollahi and Fariborz [24]. Baghestani et al. [25] solved the mixed mode problem of an