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# Sensitivity and uncertainty quantification of the cohesive crack model

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ABSTRACT

The zero thickness, fracture mechanics inspired cohesive crack model has been widely used in its various formulations. The constitutive model being formulated in terms of about fourteen parameters, yet only few can be measured experimentally, and other must be estimated.

This paper performs a sensitivity analysis to assess the relative importance of each of the parameters resulting in the model Tornado diagram. For the most sensitive parameters, uncertainty quantification is performed through Latin hypercube sampling to determine capacity and fragility curves. Finally, impact of correlation among the parameters is assessed.

The study is conducted by performing pushover analysis of a simple interface element under mode I and II, and dynamic analysis of a dam with joint elements subjected to mixed-mode fracture. This investigation leads to a probabilistic-based safety assessment of structures which responses is primarily governed by cohesive cracking.



## 1. Introduction

Zero thickness interface elements were first developed in the context of rock mechanics [1]. Cohesive crack models were in turn first proposed by [2,3]. Indeed [4] has shown that any elasto-plastic material does have a cohesive zone and does exhibit a size effect. Cohesive crack models have indeed gained much acceptance as an alternative to linear elastic fracture mechanics. As to cementitious materials, the Hillerborg's cohesive crack model [5] defined a new class of fracture mechanics-based interface elements [6–9]. They are used in the context of the so-called discrete crack model (as opposed to smeared crack model) in the finite element simulation of cracking. These finite elements will be collectively referred to as cohesive crack models subsequently. Those elements were used in the context of numerous applications in quasi-brittle materials (primarily concrete, but also rock, ceramics, stiff soil) [10,11], or through simplifications of these models in the context of blast such as [12].

The cohesive elements would typically be formulated in terms of well over ten parameters (described below). A major challenge in their use is the selection of the parameters as only few can be measured experimentally, and the remaining must be estimated. Hence, a critical question is how important is the accurate estimate of each of the model parameters. This can only be achieved through sensitivity and uncertainty analyses.

Sources of uncertainty can in be traced to one of eight groups [13]. Chief among them is the basic random variables (RVs), $\mathbf{X} = (X_1, \ldots, X_n)$. The RVs in turn can be categorized as aleatory or epistemic [13]. An aleatory uncertainty is presumed to be

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**Nomenclature**

| | |
|---|---|
| EDP | engineering demand parameter |
| ETA | endurance time analysis |
| IM | intensity measures |
| LHS | Latin hypercube sampling |
| LS | limit state |
| MCS | Monte Carlo Simulation |
| PGA | peak ground acceleration |
| POA | pushover analysis |
| RV | random variable |
| CSD | crack sliding displacement |
| COD | crack opening displacement |
| STD | standard deviation |
| COV | coefficient of variation |
| GM | ground morion |
| $S_a(\mathrm{T})$ | spectral acceleration |
| $\mathbf{X}$ | vector of basic random variables |
| $X_i$ | basic random variable |
| $c$ | cohesion |
| $\phi_f$ | angle of friction |
| $f_t$ | tensile strength |
| $\tau_1,\ \tau_2$ | tangential components of the interface traction vector |
| $\sigma$ | normal traction component |
| 2D | two-dimensional |
| 3D | three-dimensional |
| $u^{\mathrm{ieff}}$ | norm of the inelastic displacement vector |
| $u^i$ | inelastic displacement |
| $u$ | displacement |
| $u^e$ | elastic displacement |
| $u^p$ | plastic displacement |
| $u^f$ | fracturing displacement |
| $G_F^I$ | mode I fracture energy |
| $G_F^{II}$ | mode II fracture energy |
| $s_{1\sigma}$ | tensile stress at break-point |
| $w_{1\sigma}$ | COD at break-point |
| $s_{1c}$ | cohesion at break-point |
| $w_{1c}$ | CSD at break-point |
| $k_t$ | tangential stiffness |
| $k_n$ | normal stiffness |
| $\phi_d$ | dilatancy angle |
| $\gamma$ | relative irreversible deformation |
| $u_{D_{max}}$ | maximum displacement for dilatancy |
| $\Vert \cdot \Vert$ | norm operator |
| $\mathfrak{D}$ | damage parameter |
| $k_{ns}$ | secant of the normal stiffness |
| $k_{no}$ | initial normal stiffness |
| $A_o$ | total interface area |
| $A_f$ | fractured interface area |
| $n$ | number of RVs |
| $N$ | number of input parameters in model |
| $X_i^{\mathrm{mean}}$ | mean value of the *i*th RV |
| $X_i^{\mathrm{min}}$ | minimum value of the *i*th RV |
| $X_i^{\mathrm{max}}$ | maximum value of the *i*th RV |
| $\Theta^{\mathrm{Ref}}$ | reference response of sensitivity analysis |
| $\Theta_i^{\mathrm{min}}$ | response of sensitivity analysis with $X_i^{\mathrm{min}}$ |
| $\Theta_i^{\mathrm{max}}$ | response of sensitivity analysis with $X_i^{\mathrm{max}}$ |
| $\Theta_i^{\mathrm{swing}}$ | swing of the response in *i*th RV |
| $\Theta$ | response of sensitivity analysis subjected to $\mathbf{X}$ |
| $S$ | vector of RV's number |
| $D$ | distributional model |