Engineering Fracture Mechanics 155 (2016) 36–48

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

Development of a brittle fracture criterion for prediction of crack propagation path under general mixed mode loading

S.H. Sajjadi [a], D. Salimi-Majd [b], M.J. Ostad Ahmad Ghorabi [a,*]

[a] Department of Mechanical Engineering, Semnan Branch, Islamic Azad University, Semnan, Iran
[b] Young Researchers and Elite Club, Arak Branch, Islamic Azad University, Arak, Iran

ARTICLE INFO

Article history:
Received 13 September 2015
Received in revised form 9 January 2016
Accepted 11 January 2016
Available online 16 January 2016

Keywords:
Crack growth direction
Effective stress
Crack initiation angle
Stress intensity factor
Triaxial loading

ABSTRACT

To predict the crack propagation trajectory under general mixed mode loading, a novel criterion has been developed. For this purpose, the material-dependent effective stress proposed in the *maximum effective stress* criterion based on involving all effective components in crack extension under mixed mode I/II, has been developed for mixed mode I/II/III. The effects of the ratio of fracture toughness in mode III to that of mode I (K_{IIIC}/K_{IC}) on the crack growth have been considered by using a new material parameter. The predicted results of developed criterion in different mixed mode conditions show good agreement with the available experimental ones.

1. Introduction

Up to now, different criteria have been presented for the prediction of the angle of crack growth under in-plane loading (mixed modes I and II). Most of these criteria predict one specific angle of crack growth which is independent from material properties for every mode ratio. By examining the experimental results, it seems that the angle of crack growth under mixed mode conditions is affected by mechanical properties of the material, especially the ratio of fracture toughness for pure mode II to that of pure mode I. In pure mode II, the predicted angles are not the same for various criteria. Also, experimental results of the crack growth angle in pure mode II for different materials show different values in a widely dispersed form [1]. This can be due to the difference between the ratios of shear strength to tensional strength of one material in comparison to another. Not considering this issue can lead to decreased accuracy in prediction and the emergence of disagreements between the predicted results and experimental ones. Such a difference in pure mode II loading conditions, which is a specific state of loading, is completely evident [1]. In this condition of loading, the results of the existing criteria are different from one another and are considerably scattered. To solve this problem, Sajjadi et al. [1] proposed the *maximum effective stress* criterion for mixed mode I/II loading which has the capability of considering the ratio of shear strength to tensional strength of materials in predicting the angle of crack growth. By involving the shear stress component in the fracture phenomenon in equations along with a material parameter for every material, this criterion considers the effects of the differences between shear and tensional strength in predicting the angle of crack growth. This advantage has led to an increased accuracy in predicting the mentioned criterion and its agreement with experimental results. In this paper, the maximum effective stress criterion will be developed for triaxial loading (mixed mode I/II/III). Before explaining the developed criterion, the commonly used fracture criteria are introduced.

* Corresponding author.
E-mail address: mjafarostad@gmail.com (M.J. Ostad Ahmad Ghorabi).

http://dx.doi.org/10.1016/j.engfracmech.2016.01.015
0013-7944/

Nomenclature

Symbols

K_{I}	global stress intensity factor corresponding to pure mode I
K_{II}	global stress intensity factor corresponding to pure mode II
K_{III}	global stress intensity factor corresponding to pure mode III
K_{I}^*	local stress intensity factor corresponding to pure mode I
K_{II}^*	local stress intensity factor corresponding to pure mode II
K_{III}^*	local stress intensity factor corresponding to pure mode III
$K_{\mathrm{I}C}$	critical stress intensity factor pertaining to pure mode I
$K_{\mathrm{II}C}$	critical stress intensity factor pertaining to pure mode II
$K_{\mathrm{III}C}$	critical stress intensity factor pertaining to pure mode III
$K_{\mathrm{I}\theta}$	stress intensity factor of equivalent mode I for mixed mode I/II
K^*	effective stress intensity factor
r	the radial distance from crack tip
T	T stress
α	material parameter
β	material parameter
θ	arbitrary angle around the crack tip
θ_f	crack growth angle
σ_z	normal stress in the direction of the z axis in cylindrical coordinates
σ_φ	normal stress in the direction of the φ axis in cylindrical coordinates
σ_{rr}	radial stress component
$\sigma_{\theta\theta}$	tangential stress component
$\sigma_{z\theta}$	out of plain shear stress component
$\sigma_{r\theta}$	in-plain shear stress component
$\sigma_{\theta\theta,\mathrm{Mixmode}}$	tangential stress component pertaining to mixed mode
$\sigma_{\theta\theta,\mathrm{modeleq}}$	tangential stress component pertaining to equivalent mode I
σ_1'	greatest principal stress on the surface of a cylindrical domain around the crack front
$\sigma_{\theta\theta}^*$	tangential stress component of effective stress
$\sigma_{r\theta}^*$	in-plain shear stress component of effective stress
σ_{eff}^*	effective stress
$\sigma_{eff,C}^*$	critical effective stress
$\tau_{\varphi z}$	shear stress component in the φz plane in cylindrical coordinates
ψ_0	angle of principal stress with the φ direction
φ_0	perpendicular direction to the direction of maximum principal stress
υ	Poisson's ratio

As for strain-based criteria, Chang [2] proposed maximum tangential strain criterion. In this criterion, crack is considered to propagate when the tangential strain reaches its maximum value. Scheider et al. [3] used the CTOA (Crack Tip Opening Angle) to predict the crack growth. The CTOA is a crack driving force parameter which is closely related to CTOD (Crack Tip Opening Displacement) parameter. As for stress-based criteria, Erdogan and Sih [4] proposed *Maximum Tangential Stress (MTS) criterion*. This criterion considers that the crack propagates along the direction where the tangential stress has its maximum amount in a specific distance from the crack tip. The predicted results of this criterion are in a proper agreements with empirical results of brittle materials under fatigue loading [5,6]. Also this criterion has been used successfully for prediction of elliptical micro-crack growth [7]. Furthermore, this method was developed and used for anisotropic materials by Carloni and Nobile [8]. Moreover, this criterion was extended to be applicable for V-notched and U-notched specimens [9,10]. To increase the precision of the MTS criterion, William and Ewing [11], Aliha and Ayatollahi [12], Hadj Meliani et al. [13], Cheng et al. [14], Mirsayar [15] and Ayatollahi and Saboori [16] took the nonsingular stress term, namely T stress term, into account and extended the MTS criterion into the *Generalized Maximum Tangential Stress* (GMTS) criterion. Moreover, Goldstein and Salganik [17] introduced the principle of local symmetry which assumes that crack propagates along the path where mode II stress intensity factor (K_{II}) vanishes. In order to consider the effects of other stress components in crack growth direction, Sajjadi et al. [1] proposed the effective stress criterion for mixed mode I/II. Schöllmann et al. [18] proposed the *Maximum Principal Stress (MPS)* criterion which considers that crack propagates along the direction of maximum principal stress around the crack front in a general triaxial stress state. For plane-stress, this criterion correlates to the MTS criterion [19,20]. Although this criterion could be applied for mixed modes I/II/III, it predicts a unique angle for a certain mixed mode ratio for all materials. Su and Cui [21] and Khan and Khraisheh [22], by using Von Mises as the yielding criterion, developed a modified maximum circumferential stress criterion considering the maximum hoop (tangential) stress at the boundary between plastic and elastic zones. Hussain et al. [23] proposed *G criterion* with considering this assumption that the crack