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A polynomial chaos method for the analysis of the dynamic behavior of uncertain gear friction system

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ABSTRACT

In this paper, we propose a new method for taking into account uncertainties occurring due to gear friction, based on the projection on polynomial chaos. The new method is used to determine the dynamic response of a spur gear system with uncertainty associated to friction coefficient on the teeth contact. The simulation results are obtained by the polynomial chaos method for dynamic analysis under uncertainty. The proposed method is an efficient probabilistic tool for uncertainty propagation. The polynomial chaos results are compared with Monte Carlo simulations.

1. Introduction

The study and analysis of the dynamic behavior with nonlinear systems is a major interest in the industrial sector. Thus they allow overcoming the areas of instability and reducing vibration levels. Indeed, the negative consequences that may result from the instability of systems require designers to develop the most rigorous solution. This passes through a detailed study and analysis of the dynamic behavior of these systems before considering their actual implementation.

Several parametric studies have shown the great sensitivity of the dynamic behavior of gear systems. However, these parameters admit strong dispersions. Therefore, it becomes necessary to take into account these uncertainties to ensure the robustness of the analysis (Guerine et al., 2015a, 2015b). Also there are several studies in reliability for vibration structures taking into account the uncertainties (Abo Al-kheer et al., 2011; Mohsine and El Hami, 2010; El Hami et al., 2009; Radi and El Hami, 2007; El Hami and Radi, 1996; El Hami et al., 1993).

The mechanisms of transmission by gear tooth contact are characterized by the presence of friction coefficient that affects the vibration and noise of these systems. Parameter estimation is an important problem, because many parameters simply cannot be measured physically with good accuracy, such us the friction coefficient, especially in real time application (Blanchard et al., 2009, 2010).

The coefficient of friction is a very important factor for designing, operating, and maintaining the gear transmission. Indeed, the accurate estimation of this coefficient is difficult due to the effects of various uncertain parameters, e.g., materials of gears, roughness and contact patch size, etc. However, the friction coefficient admits a strong dispersion (Nechak et al., 2011; Lee et al., 2012). Therefore, it becomes necessary to take into account these uncertainties in order to ensure the robustness of the analysis. A study of the nonlinear dynamic behavior will help to analyze stability and to predict the vibration levels according to the parameters variations.

Several methods are proposed in the literature. Monte Carlo (MC) simulation is a well-known technique in this field (Fishman, 1996). It can give the entire probability density function of any system variable, but it is often too costly since a great number of samples are required for reasonable accuracy. Parallel simulation (Papadrakakis and Papadopoulos, 1999) and proper orthogonal decomposition (Lindsley and Beran, 2005) are some solutions proposed to circumvent the computational difficulties of the MC

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method.

Polynomial Chaos Expansion (PCE) is presented in the literature a more efficient probabilistic tool for uncertainty propagation. It was first introduced by Wiener and launched by Ghanem and Spanos who used Hermite orthogonal polynomials to model stochastic processes with Gaussian random variables (Li and Ghanem, 1998).

Polynomial Chaos (PC) gives a mathematical framework to separate the stochastic components of a system response from the deterministic ones. It used to compute the deterministic components called stochastic modes in an intrusive and non-intrusive manner while random components are concentrated in the polynomial basis used. The Polynomial Chaos (PC) method has been shown to be considerably more efficient than Monte Carlo in the simulation of systems with a small number of uncertain parameters (Blanchard et al., 2009; Sandu et al., 2006).

The main originality of the present paper is that the uncertainty of the gear friction system in the dynamic behavior study of one stage gear system is taken into account. The main objective is to investigate of the capabilities of the new method to determine the dynamic response of a spur gear system subject to uncertain friction coefficient. So, an eight degree of freedom system modelling the dynamic behavior of a spur gear system is considered. The modelling of a one stage spur gear system is presented in Section 2. The modelling of friction coefficient is presented in Section 3. In the next section, the theoretical basis of the polynomial chaos is presented. In Section 5, the equations of motion for the eight degrees of freedom are presented. Numerical results are presented in Section 6. Finally in Section 7, to conclude, some comments are made based on the study carried out in this paper.

2. One stage spur gear system modelling

The dynamic model of the one stage gear system is represented on Fig. 1. This model is composed of two blocks. Every block is supported by flexible bearing which the bending stiffness is k_1^x and the traction-compression stiffness is k_1^y for the first block, k_2^x and k_2^y for the second block, respectively.

The wheels (11) and (22) characterize the drive and the respectively the driven gears. The two shafts (1) and (2) admit some torsional stiffness k_1^θ andk_2^θ.

Angular displacements of every wheel are noticed by $\theta_{(1,1)}$, $\theta_{(1,2)}$, $\theta_{(2,1)}$ and $\theta_{(2,2)}$. Besides, the linear displacements of the bearing noted by x_1 and y_1 for the first block, x_2 and y_2 for the second block, are measured in the plan which is orthogonal to the wheels axis of rotation (Kahraman et al., 2007).

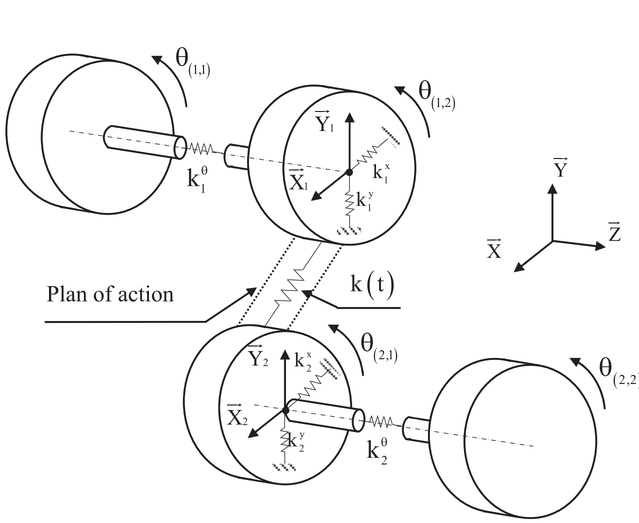

Fig. 1. Model of the one stage gear system.

Fig. 2 defines a reference frame $(O, \overrightarrow{X}, \overrightarrow{Y})$ and the position of the wheels of the one stage gear system.α is the pressure angle of gearmesh contact.

The teeth deflection is expressed along the line of action, and it can be written as:

$$\delta(t) = s^\alpha(x_1 - x_2) + c^\alpha(y_1 - y_2) + r^b_{(1,2)}\theta_{(1,2)} - r^b_{(2,1)}\theta_{(2,1)} \tag{1}$$

where $s^\alpha = \sin(\alpha)$ and $c^\alpha = \cos(\alpha)$ and $r^b_{(1,2)}$,$r^b_{(2,1)}$ represent the base gears radius.

Generally the gear mesh stiffness variation k(t) is modelled by a sinusoid wave or by a square wave form. The later is the most representative of the real phenomenon and is represented on Fig. 3. The gear mesh stiffness variation can be decomposed in two components: an average component noted by kc, and a time variant one noted by kv(t) (Walha et al., 2009).

$$k(t) = kc + kv(t) \tag{2}$$

The time component of the mesh stiffness is defined by the following periodic form (Fig. 4).

The extreme values of the mesh stiffness variation are defined by:

$$k_m = \frac{kc}{2\varepsilon^\alpha} \quad \text{and } k_M = -k_m\frac{2-\varepsilon^\alpha}{\varepsilon^\alpha - 1} \tag{3}$$

ε^α and Te represent respectively the contact ratio and mesh period corresponding to the two gearmeshes contacts.

3. Friction coefficient modelling

The instantaneous coefficient of friction is defined as the ratio between the measured friction force F_f and the normal force F_n. In the case of the gear system, the number of components of the friction force is equal to the number of pair of teeth in contact. The modeling of the friction forces is usually made based on the Coulomb law. According to this model the friction coefficient is assumed constant. In the dynamic model, the friction can be introduced by two friction torques applied on the gears (12) and (21) (Fig. 5).

On the first spur gear (12), the friction torque is expressed by:

$$Cf_{12}(t) = F_f(t).\xi_1(t) \tag{4}$$

Also, on the second spur gear (21), the friction torque is expressed by:

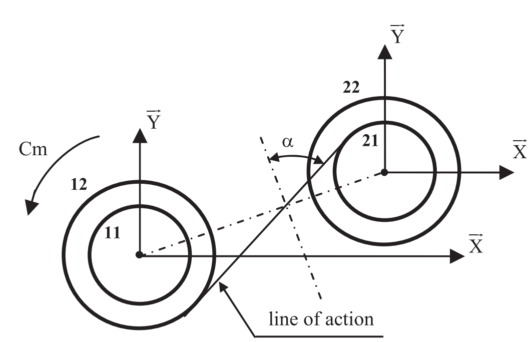

Fig. 2. Position of the wheels of the one stage gear system.