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Size dependency in vibration analysis of nano plates; one problem, different answers

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A B S T R A C T

In addition to the classical (local) case, several non-classical constitutive equations consisting first order strain gradient (five versions), second order strain gradient (four versions), differential form of non-local and implicit gradient (IG) were employed to model the vibrational behavior of plates. Employing these models, vibration governing differential equations were developed. In addition to the classical material characteristics, a non-classical property is interfered into the equations. To solve the PDEs under two different boundary conditions, Navier approach was employed to obtain parametric answers for the plate natural frequencies. Moreover, Galerkin approach was employed to solve the PDEs numerically. The later approach was also applied to obtain approximated parametric answers. The results obtained via the introduced theories in the current research and the relevant ones reported in the literature were compared. According to the obtained parametric answers, the roles of different parameters on the natural frequencies were explained. In the part of numerical studies, influence of different parameters consisting classical and non-classical properties, boundary conditions and the plate size on the natural frequencies were studied.

1. Introduction

Small size structures such as micro and nano beams and plates are used in Micro and Nano Electro Mechanical Systems (MEMS/NEMS) as actuators and sensors (Baughman et al., 1999; Wilson et al., 2007). For example: application of micro beam as an essential part of AFM (Wilson et al., 2007), employing single-layered graphene sheets (SLGS) as mass sensors, atomistic dust detector and strain sensor (Sakhaee-Pour et al., 2008a,b) and many other examples (Borisenko et al., 2001)[5]. Literature review indicates that GS has high potential applications in various fields of technology so that it can be considered as a forehand nano structure. In this regard, more scientific investigation is necessary to understand the properties of this element and prediction of its behavior at different situations. Indeed, the more realistic modeling of such elements the more reliable predictions of the behavior of their performance is provided under different working conditions. This is a strong motivation for the researchers to investigate on this subject.

In the field of mechanics, vibration analysis is one of the vital studies for almost all structures, especially for beams and thin plates. Following such study, natural frequencies of the structure is determined, as a requirement for control process in dynamic applications. In fact, relation of the natural frequencies to the structure size, properties and boundary conditions are interested in such studies. Due to the difficulty in fabrication of nano structures at certain sizes and implementation of experimental setup and applying boundary conditions, more investigation on the theoretical approaches is required to understand the concept and revealing possible correlations. In this regard, two categories of theoretical studies consisting molecular dynamics (MD) and continuum based approaches can be followed for the mentioned purpose. Although MD method can be employed in this field, it does not provide a parametric viewpoint to the problem; and to understand each parameter dependency, several computer runs should be done which might take long time. On the other hand, continuum based approaches are good candidate to model the nano structures providing possibility of parametric studies. However, there is a main question about the adequate formulations to capture the right behavior of nano structures. In other words, there is a main question about constitutive equation to model the relation between the

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influencing variables e.g. stress' and strains.

To develop the governing equations in vibration analysis of the plates, equilibrium equations, kinematic relations of the plate displacement and material constitutive equations are required. The well-known Newton's equilibrium equations can be applied to nano structures similar to large ones. In terms of the kinematic relations, a few approaches have been employed to model the plate deformation mathematically such as classical plate theory (Kirchhoff) and higher order shear formulation (Mindlin theory) (Venstel and Krauthammer, 2001). Due to the plate thickness, Kirchhoff thin plate theory is expected to capture the deformation behavior of SLGS adequately, especially when only the few lower natural frequencies are under determination.

As mentioned, constitutive relations representing the material behavior is another requirement to develop the governing equations of the plate vibration. If the classical local theory (e.g. Hook's law) is employed at this step, no difference is seen between the final governing equations of large and nano scale structures, because there is no parameter representing the size effects in the hierarchy of the problem formulation. Nevertheless, there are many atomistic simulations and experimental evidences showing that the material characteristics are changed dramatically when the smallest size of the structure approaches to the order of nano in various categories of media such as ductile metals (Fleck et al., 1994; Poole et al., 1996), brittle materials (Vardoulakis et al., 1998), polymers (Chong and Lam, 1999; McFarland and Colton, 2005) and polysilicon (Chasiotis and Knauss, 2003). However, it has been proved that the properties are related to the structure morphology and size but no threshold for the changes is reported (Chasiotis and Knauss, 2003; Chong and Lam, 1999; Fleck et al., 1994; McFarland and Colton, 2005; Poole et al., 1996; Vardoulakis et al., 1998). In terms of morphology dependency, for instance, properties of a single layer nano structure in tubular form are different from those for the planar one, although both are made of identical atoms. Moreover, for a specific morphology such as nano tube, the diameter and length affect the material characteristics (Jafari et al., 2012). As another important morphology, SLGS, no unique agreement is seen about size dependency of its behavior in the open literature. Employing MD approach, there are several reports showing that while keeping the thickness constant, the properties are related to the sheet's width and length so that it becomes stiffer while it is enlarged approaching some asymptotic values (Giannopoulos et al., 2011; Wang et al., 2014). Nevertheless, there are some data, following MD method, indicating no significant size dependency for that structure (Ni et al., 2010). On the other hand, using the same method for the same structure, it has been proved that a few elastic constants decreases when the SLGS size increases up to 2 nm and afterwards it takes increasing trend going to asymptotic values (Giannopoulos et al., 2011). Existence of such discrepancies between the reported data in the literature might be due to lack of verifying experiments for such small structures, which in turn is due to difficulties and complications in such studies. For instance, it is difficult to fabricate graphene sheets in certain sizes and apply definite boundary conditions to evaluate reliable experimental results. It seems that nonexistence of enough realistic data can be a reason of perfect validation to the theoretical methods leading to disagreement in the theoretical methods. Later on, this point is more illustrated.

As a consequence of the aforementioned explanation, in case of applying continuum based approaches for modeling nano structures, the constitutive equations should be architected in such a way that the role of structure size is taken into account. Compared to the well-known Hook's law which relates stresses and strains at the same point by means of stiffness constants regardless of the structure size, a size dependent formulation should contain additional parameter(s) representing size dependency. The authors believe that similar to the stiffness constant which is an inherent material property, the size effect parameter(s) can be an intrinsic characteristic of the structure which is called material length scale or non-local parameter in the literature (Eringen, 2002). Literature review indicates that a few theories considering size effects such as Eringen's integral non-local and differential non-local (Eringen, 2002), strain gradient (Mindlin and Eshel, 1968; Mindlin and Tiersten, 1962; Toupin, 1962), modified strain gradient (Lam et al., 2003), couple stress (Hadjesfandiari and Dargush, 2011), stress gradient (Aifantis, 2003, 2011) and surface energy (Dingreville et al., 2005) have been developed.

Ke and co-workers employed modified couple stress theory in conjunction with classical thin plate theory for free vibration analysis to examine the influences of the size effects, side-to-thickness ratio and aspect ratio, on the natural frequencies of the micro plate (Ke et al., 2012). They found that the natural frequency increases with increase of material length scale. Akgöz and Civalek studied the free vibration of simply supported SLGS resting on an elastic matrix by using the modified couple stress constitutive equations and thin plate kinematic relations (Akgöz and Civalek, 2012). They concluded that natural frequencies increase significantly when the material length scale is increased. Yan and Jiang employed Kirchhoff plate theory and the constitutive equations based on the surface energy to study the impact of surface effects on the vibration and buckling of a fully clamped piezoelectric nano plate (Yan and Jiang, 2012a,b). They showed that the surface effects on the natural frequencies become more important when the thickness becomes smaller. Yin and co-workers employed the modified couple stress formulation plus Kirchhoff plate kinematic theory to capture the influence of size effect on the lowest two natural frequencies of micro plates (Yin et al., 2010). Liu and co-workers studied the free vibration of simply supported rectangular nano plates using the nonlocal constitutive equations and Kirchhoff kinematic relations (Liu et al., 2013).

Indeed, the size dependent constitutive equations including those employed in the above mentioned references; provide extra coefficients as a non-classical properties or non-local parameters, representing the size effects explicitly or implicitly in addition to the classical elastic constants. Nevertheless, based on the review of the results reported in the literature, due to the studies done by the current authors and following a few criticisms reported in the literature, no absolute agreement is seen about the results of these theories while applied on an identical problem (Aifantis, 2011; Challamel and Wang, 2008; Lim, 2010; Lim et al., 2012; Taghavi and Nahvi, 2013). Moreover, in some cases, one theory predicts different behavior for the same problem under two different loadings. For example, applying the simplified differential form of Eringen's non-local theory (SDNL) (Eringen, 2002) in modeling beams, it is predicted that the buckling load has an indirect relation to the small size factor (non-local parameter) meanwhile the deflection of the same beam under transversal loading has a direct relation to this parameter (Challamel and Wang, 2008; Lim, 2010; Taghavi and Nahvi, 2013). This means that based on this theory, non-local parameter causes opposite effects of softening and stiffening under buckling and lateral loading conditions, respectively for the same beam. Employing this theory leads to critical buckling load and natural frequency of nano plates and nano beams that decrease with the increasing of the non-local parameter (Arani et al., 2012; Babaei and Shahidi, 2011; Narendar and Gopalakrishnan, 2011; Pradhan and Murmu, 2009). In other words based on the buckling/vibration study, this theory predicts that the beam becomes softer when its size decreases (Arani et al., 2012; Babaei and Shahidi, 2011; Narendar and Gopalakrishnan, 2011; Pradhan and Murmu, 2009) whereas, based on the strain