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A simplified cyclic plasticity model for calculating stress-strain response under multiaxial non-proportional loadings

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A B S T R A C T

Given the dependence of many fatigue damage parameters on both stress and strain values, being able to produce accurate stress-strain estimations is an essential first step when performing a fatigue life analysis. However, more advanced cyclic plasticity models, capable of capturing complex material behavior under multiaxial non-proportional loadings, often require the determination of a large number of material constants from experimental data that are not always available to the analyst. Therefore, in this study, a simplified Armstrong-Frederick-Chaboche style plasticity model, based on the assumption of Masing material behavior and a newly proposed transient hardening formulation, was evaluated in terms of its ability to predict stress-strain response under complex multiaxial loading conditions. To do this, experimental stress-strain data were generated for 2024-T3 aluminum alloy using both constant and variable amplitude multiaxial block loading histories designed to reflect a wide variety of stress states that might be encountered in any given service loading history. Stress-strain predictions were performed both with and without the consideration of non-proportional hardening effects in order to provide a baseline for evaluating the results. Although all stress-strain predictions were found to agree well with the experimental data, both quantitatively and qualitatively, it was shown that, in general, predictions for axial stress amplitudes and mean axial stresses were consistently better than those for shear stress.

1. Introduction

Being able to accurately estimate the stress-strain response of a material is an essential first step in making reliable fatigue life predictions. This is especially true for complex multiaxial loading conditions where many fatigue damage parameters, including popular critical plane approaches, rely on both stress and strain quantities to compute fatigue damage. For general multiaxial stress states and/or variable amplitude loading histories, phenomena such as Bauschinger effect, cyclic hardening/softening, non-proportional hardening, and material memory effect may all need to be considered. Given the abundance of engineering components and structures subjected to multiaxial variable amplitude service loadings, the practical significance of such a problem is clear.

Over the years, cyclic plasticity models have emerged as a key tool in estimating nonlinear material constitutive behavior. Although many types of cyclic plasticity models have been proposed (e.g. overlay, single surface, two-surface, multi-surface, endochronic, etc.), single yield surface models, such as those based on Armstrong-Frederick style nonlinear kinematic hardening rules, have become popular due to their robustness in cases involving both proportional and non-proportional loadings (Jiang, 1993; Chaboche et al., 1991; Jiang and Kurath, 1996). This is especially true following efforts by Chaboche et al. (Chaboche et al., 1979; Chaboche, 1987) to improve the original Armstrong-Frederick model (Armstrong and Frederick, 1966) by expressing the hardening rule as a series expansion of backstress components, each taking the same form as the original relation. Since the development of the Chaboche model, many other nonlinear kinematic hardening formulations have been proposed with the primary goal of obtaining better ratcheting prediction under complex loading conditions (Halama et al., 2012). However, most of these are still based on of the superposition of several backstress components, but with modifications of the original form.

Additionally, in order to consider the effects of loading non-proportionality on material constitutive behavior, several parameters have been proposed to relate the stress state and/or loading path to the degree of non-proportional hardening experienced by a

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Nomenclature

A	Tanaka's non-proportionality parameter
$b^{(i)}$	cyclic hardening rate constants
$\tilde{\boldsymbol{C}}$	Tanaka's non-proportionality tensor
$c^{(i)}$	kinematic hardening constants
C_c	microstructural transformation rate constant
h	equivalent plastic modulus
$H^{(i)}$	slopes of discretized stress-strain curve
k	pure shear yield stress
$\tilde{\boldsymbol{L}}^{(\boldsymbol{i})}$	unit vector of ith backstress term
M	number of backstress components
m	non-proportional hardening rate constant
$\tilde{\boldsymbol{n}}$	exterior unit normal to yield surface
N_s	number of cycles to reach stabilized material response
p	equivalent plastic strain
q	size of strain memory surface
$r^{(i)}$	transient kinematic hardening parameters
r_m	mean specimen radius
$R_P^{(i)}$	cyclic kinematic hardening constants
$R_N^{(i)}$	non-proportional kinematic hardening constants
$\tilde{\boldsymbol{S}}$	deviatoric stress tensor
T	applied torque
t	specimen wall thickness
$\tilde{\boldsymbol{\alpha}}$	total backstress tensor
$\tilde{\boldsymbol{\varepsilon}}^{\boldsymbol{p}}$	plastic strain tensor
ε_a^p	uniaxial plastic strain amplitude
$\Sigma\varepsilon_t^p$	total plastic strain incurred before material stabilization
τ	shear stress
$\chi^{(i)}$	ratcheting rate constants

material. These methods fall into two major groups: empirical formulations and constitutive models. Empirical relations, such as those proposed by Kanazawa et al. (Kanazawa et al., 1979), Itoh et al. (Itoh et al., 1995), and Kida et al. (Kida et al., 1997), relate the shape of a particular load path to a factor of non-proportionality. This factor of non-proportionality is then used, in conjunction with the material's non-proportional cyclic hardening coefficient, to modify the stress-strain response of the material. It was shown by Shamsaei et al. (Shamsaei et al., 2010), however, that empirical methods can significantly over-estimate stress response when the proportional straining direction is only gradually changed, and they cannot properly account for the progression of cross hardening when the proportional straining direction is suddenly changed.

Constitutive models, on the other hand, which are generally considered within the context of cyclic plasticity models, use continuum mechanics concepts to relate the stress-strain response of a material to its internal state throughout a given loading path. As a result, they are better able to capture the effect of more complex loading events on changes in material constitutive behavior. Examples of such models include those proposed by McDowell (McDowell, 1985), Benallal and Marquis (Benallal and Marquis, 1987), Fan and Peng (Fan and Peng, 1991), and Tanaka (Tanaka, 1994). Of these, Tanaka's parameter has been shown to be one of the more robust options available (Shamsaei et al., 2010; Jiang and Kurath, 1997; Jiang and Zhang, 2008).

Given their agreement demonstrated with experimental results in literature, the overall goal of this study was to evaluate the ability of an Armstrong-Frederick-Chaboche style cyclic plasticity model, along with Tanaka's non-proportionality parameter, in predicting stress-strain response under more complex multiaxial loading situations. To accomplish this, experimental stress-strain data were first generated using both constant and variable amplitude multiaxial block loading histories. These histories were designed to include a variety of axial dominated, shear dominated, in-phase, out-of-phase, and asynchronous multiaxial loading conditions, along with mean stresses, overload effects, and varying degrees of cross hardening and non-proportional hardening. Cyclic ratcheting and mean stress relaxation, however, were not topics of investigation in this study.

Experimental results were then compared to predictions to allow for evaluation of the model in terms of both qualitative and quantitative agreement. More advanced cyclic plasticity modeling formulations, however, while capable of capturing material behavior under such complex loading conditions, often require the determination of a large number of material constants from experimental data. Therefore, since this study is concerned with plasticity modeling for application to fatigue life analysis, where usually only basic information on material deformation behavior is available, an emphasis was placed on developing a simplified version of the plasticity model. This way, all necessary material constants could be determined from a relatively limited amount of experimental data.

Although the work presented herein focuses mainly on steady-state material deformation behavior, transient effects must still be considered due to changes in loading non-proportionality. As a result, a simplified transient hardening formulation was proposed, along with new calculation methods for determining the related material constants. Consequently, making sure that the model was able to properly account for transient non-proportional hardening was a major emphasis of this study. Extension of the modeling procedures to general cyclic hardening behavior is currently ongoing and will be presented in a future publication.

2. Cyclic plasticity model

2.1. Model description

The basis for the plasticity model evaluated in this study was a slightly modified formulation of the Armstrong-Frederick-Chaboche style single surface nonlinear kinematic hardening rule proposed in (Ohno and Wang, 1994; Jiang and Sehitoglu, 1996a). This particular formulation was chosen due to its ability to accurately model stress-strain behavior under a variety of loading conditions, as demonstrated in (Jiang and Kurath, 1996, 1997; Shamsaei et al., 2010; Jiang and Sehitoglu, 1996b; Zhang and Jiang, 2008). The model assumes time-independent and isothermal response of a homogeneous and initially isotropic material. A von Mises yield function was used to describe yielding conditions as deformation tests results, shown in Fig. 1, demonstrated good correlation between uniaxial and multiaxial stress-strain behavior using this criterion. Additionally, to account for the effects of non-proportionally varying stresses, Tanaka's non-proportionality parameter was implemented into the model.

The hardening rule assumes the series expansion of backstress components originally introduced by Chaboche:

$$\tilde{\boldsymbol{\alpha}} = \sum_{i=1}^{M} \tilde{\boldsymbol{\alpha}}^{(i)} \tag{1}$$