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Thermal stress analysis in metallic plates with a non-circular hole subjected to uniform heat flux

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ABSTRACT

The purpose of this study is to evaluate stress distribution around a non-circular hole in an infinite metallic plate subjected to a uniform heat flux. The development of the Goodier and Florence's method is used for the stress analysis of perforated plates subjected to a uniform heat flux. Goodier and Florence used their solution for the stress analysis of infinite isotropic plates with circular and elliptical holes. In order to expand their method to study non-circular holes, by means of conformal mapping, the infinite area external to the hole can be presented by the area outside the unit circle. In this study, thermal-insulated condition along the hole boundary is assumed. The rotation angle of hole, bluntness and the angle of heat flux as important parameters are considered in this study. Results showed that the effects of these parameters on stress distribution around various shaped holes are very significant and by the correct selection of these parameters, a lower amount of thermal stress compare to that of a circular hole can be achieved.

1. Introduction

Plates are widely used in industry. They are considered as the main components of structures operating at high temperatures. Plates are used in the design of nuclear reactors, new steam and gas turbines and jet and racket engines. Components of such structures are subjected to the thermal loading. When a uniform heat flux at a solid body is disturbed by the presence of geometric discontinuities such as cracks or holes, thermal stresses are induced. These stresses cause the premature failure. Consequently, the useful life of engineering structures will be reduced. Several studies have been conducted to evaluate the stress distribution around holes in perforated plates under a uniform heat flux.

Florence and Goodier (1955) and Muskhelishvili (1963) developed the basic theory of thermo-elasticity in which complex variable method for stress analysis of perforated plates was used. Using the complex variable method, Florence and Goodier (1960) obtained the thermal stress for an isotropic elastic material containing a circular and an ovaloid hole. General relations for the thermo-elastic problem of an infinite elastic and isotropic plate with a hole of arbitrary shape were described by Gaivas (1966). As a special case, he considered the equilibrium of an elastic medium with an elastic inclusion in the form of an elliptical cylinder. By using the complex variable method and complex stress function, Takeuti and Sumi (1976) obtained the thermal stress distribution in finite plate with a central circular hole. Nisitani et al. (1991) presented a solution for calculating the thermal stress in an infinite plate with a circular hole Under uniform heat flux. The thermo-elastic problems of discs and holes or inclusions in an infinite isotropic plate were studied by Kattis (1991). The boundary of the problems was mapped onto the unit circle by using a suitable conformal mapping. The Solution of two-dimensional thermo-elastic problems for the perforated plates was fully described by Hasebe and Wang (2005). They utilized the complex variable method for the stress analysis of isotropic plates with a hole under uniform heat flux. Complex potential functions were derived for different mechanical and thermal boundary conditions. The general solutions for the external force, displacement, and mixed boundary value problems under both the uniform heat flux and a point heat source were separately described. Using an elliptic coordinate system, Bhullar (2006) calculated the thermal stresses and temperature distribution in the polygon regions containing an elliptical hole. In this study, it was assumed that the region is thermally insulated at the outer boundary with an internal convective boundary and is free from an external force. Thermo-elastic solution to a coated elliptic hole embedded in an infinite plate under

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uniform heat flux was investigated by Chen and Chao (2008). Aseeri (2008) used the complex variable method to achieve the potential and Gaursat functions for an infinite elastic plate weakened by an arbitrary shaped hole. In order to achieve an explicit solution, the curvilinear hole was conformally mapped outside a unit circle by means of a rational mapping function. Assuming the plane stress conditions, Bhullar and Wegner (2009) discussed the thermal stress analysis of isotropic plate with a hole in a hyperelliptical shape. The complex variable method was used to calculate stress fields under isothermal conditions. Yoshikawa and Hasebe (1999) obtained the Green's functions for an infinite isotropic plate with an arbitrary shaped hole under adiabatic and isothermal boundary conditions with a heat source. Hasebe et al (Hasebe, 2010). presented stress analysis for soft ferromagnetic, paramagnetic, and diamagnetic materials of an infinite thin plate with an elliptical hole subjected to a steady electric current. By means of a rational mapping function, Hasebe et al. (2010) analyzed the temperature, heat flux and thermal stress induced by an electric current for a thin infinite conductor containing an elliptical hole with an edge crack. Considering the solution of crack problem in Hasebe's studies, the results were revealed as a closed form. Further, the stress intensity factor was calculated. Additionally, the interaction problem between a cracked hole and a line crack was investigated by Vinh et al. (2005). The problem was solved using the complex variable method and the Green's function. Based on the complex variable method and using a rational mapping function, Hasebe et al. (2007) investigated infinite plate containing a rigid inclusion interacting with a line crack subjected to a uniform heat flux. In this study, the rigid body rotation of the inclusion was considered. In a recent paper by the authors, the effect of rotation angle of hole and, bluntness on stress distribution around a quasi-rectangular hole in an infinite isotropic plate taken into consideration. Thermal-insulated condition along the hole boundary was assumed. Also, the complex variable method and conformal mapping were used (Jafari et al., 2015). Recently, Chen (2015) presented a general solution for an elliptical inclusion in plane steady thermo-elasticity based on the complex variable method and the conformal mapping technique. Moreover, Zhang and Wang (2016, using the conformal mapping technique and the continuity conditions along the interface, investigated an elliptical hole or a crack in an infinite two-dimensional thermoelectric media subjected to remote uniform electric current density and energy flux.

Despite a large research effort, no study has been completely conducted the evaluation of the thermal stresses induced by a uniform heat flux in an infinite elastic medium containing a hole with various shapes. In the present study, by using the complex variable method, an attempt has been made to show the influence of key parameters such as rotation angle of hole and bluntness on the stress distribution around regular holes with various shapes.

2. Problem description and material

An infinite perforated plate subjected to the uniform heat flux, q, in an arbitrary direction with respect to the x-axis, δ, is considered in this study. A non-circular hole is located at center of plate (Fig. 1). The hole size is small enough, that its effect will be negligible at a distance of a few diameters from its edge. The material of the plate is assumed to be isotropic, homogeneous and satisfy the generalized Hooke's law. The hole can take arbitrary orientation such that the major axis of the hole is directed at angle β with respect to horizontal axis, as shown in Fig. 1. Stress free and thermal insulated conditions along the hole boundary are considered.

For an isotropic plate with hole, the normalized stress which is earlier defined in this paper is independent of material properties. Therefore, the results of this research can be used for isotropic plates with other mechanical properties. The material properties used in this research are listed in Table 1.

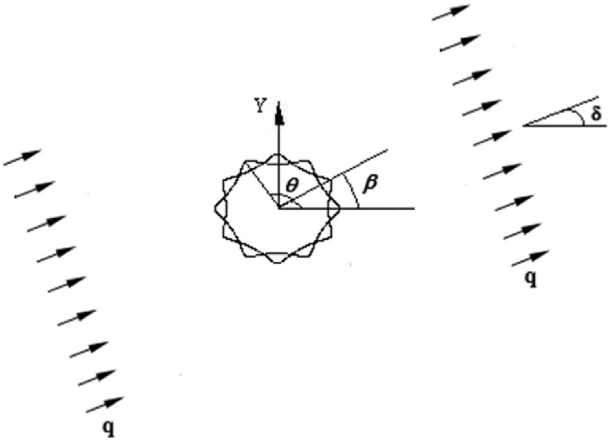

Fig. 1. Infinite plate under uniform heat flux q.

3. Basic relations

Florence and Goodier (1955, 1960) presented the stress distribution around circular and elliptical hole in an isotropic infinite plate under the uniform heat flux. In order to extend this method to other holes with non-circular shapes, establishing a relation between any holes and a circular hole is necessary. A conformal transformation can be used to map the area external to a non-circular hole in z-plane into the area outside the unit circle in ξ-plane (Fig. 2). Such a mapping function is as follows: (Jafari and Ardalani, 2016):

$$z = \omega(\xi) = x + iy \tag{1}$$

In the above equation, x and y obtained as follows (Jafari and Ardalani, 2016).

$$\begin{aligned} x &= \lambda(\cos\theta + w\cos n\theta) \\ y &= \lambda(\sin\theta - w\sin n\theta) \end{aligned} \tag{2}$$

The parameter λ, which is a positive and real number, controls the size of the hole. Integer n determines the number of the hole sides. The hole sides are given by $n+1$. The radius of curvature at the corner of the hole changes by changing the parameter w. The conditions $0 \le w < 1/n$ ensure that the hole shape does not have loops. Fig. 2 represents the various shapes of the hole that are created by the Equation (2). The x and y coordinates are shown in Fig. 3 for quasi-square hole for different parameters w.

It is well known that, for the plane stress problems, all stress components can be considered in terms of a single stress function $F(x,y)$. By substituting the stress function into the compatibility equation, a fourth order differential equation in F is obtained.

$$\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0 \tag{3}$$

By using the complex variable method, Muskhelishvili (1963)

Table 1
Material properties (Fisher, 2005).

Material	α_0(μm/m/°C)	k(W/m°C)	E(MPa)	ν
Steel	11.1	50.2	210	0.3