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# Real-time prognosis of random loaded structures via Bayesian filtering: A preliminary discussion

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ABSTRACT

Particle filters are effective tools for the monitoring of damage propagation phenomena. However, a common hypothesis of particle filters for damage prognosis is the constant-amplitude fatigue loading affecting the damage growth. This work constitutes a preliminary analysis of the performance of particle filtering in case of random load, relaxing the hypothesis of constant-amplitude fatigue. Two case studies referring to stationary random loads are introduced: the first concerning a narrow-band stress history, while the second focusing on a wide-band stress spectrum. A solution for each case study is provided and validated using numerical simulations of fatigue cracks in a metallic plate.


## 1. Introduction

Bayesian filtering methods prove to be powerful tools for the monitoring and prediction of degradation processes. Especially in the field of structural degradation, their ability to explore and filter the state-space of the system allows a reliable and fast estimation of the remaining useful life (RUL) of the structure. The Bayesian updating of the RUL can be performed by means of traditional non-destructive techniques or real-time structural health monitoring systems that provide new information on the degradation state. Among several Bayesian filtering algorithms, particle filter (PF) (also known as sequential Monte Carlo sampling, or sequential importance sampling/resampling) is particularly suitable for degradation processes because of its capability to describe nonlinear systems affected by non-Gaussian probability density functions (PDFs) of the system variables or model parameters. It has been applied to fatigue-induced degradation processes frequently showing promising results. Among the most relevant applications of PF for structural integrity assessment, Orchard et al. applied PF algorithms to monitor and predict fatigue crack propagation of a turbine engine as well as a planetary carrier plate [1–3]. Other applications of PF for fatigue crack growth (FCG) can be found in [4–7]. All of them grounded on the well-known Paris' law [8] for the estimation of the FCG rate defined as length per load cycle. PFs grounding on the NASGRO model [9] are available in [10,11]. Focusing on PF for the prognosis of composite materials, several works have been recently published by Chiachio et al. [12–14], to name just a few. In these papers, the authors made use of a modified Paris' law based on the strain energy release rate to estimate the matrix crack density growing into cross-ply laminates. They predict the evolution of the matrix crack density as well as the stiffness reduction of the material, and these predictions were combined to estimate the RUL of carbon fiber-reinforced polymer coupons. In [14], they used a PF algorithm with the technique of subset simulation to enhance the prognostic capabilities. Instead, the work in [15] shows the application of PF for creep problems. All of the cited publications try to solve one or several specific problems of the real-time prognosis of

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**Nomenclature**

| | |
|---|---|
| $C$ | empirical parameter of the fatigue crack growth rate model |
| $d$ | parameter of the measurement system |
| $f_r,\ f_{r,1},\ f_{r,2}$ | stress history frequencies |
| $F$ | crack shape function |
| $k$ | discrete time index |
| $K$ | stress intensity factor |
| $m$ | empirical parameter of the fatigue crack growth rate model |
| $N$ | load cycle |
| $N_s$ | number of particles |
| $N_f$ | failure cycle |
| $t$ | time |
| $u$ | input of the system |
| $R$ | stress ratio |
| $S$ | stress |
| $S_0$ | mean stress |
| $S_a$ | stress amplitude |
| $w$ | weight of the particles |
| $x$ | state variable (semi-crack length) |
| $x_0$ | initial semi-crack length |
| $x_{CR}$ | critical semi-crack length |
| $z$ | observation of the system state |
| $\beta_0,\ \beta_1$ | parameters of the measurement system |
| $\gamma$ | empirical parameter of the fatigue crack growth rate model |
| $\delta_{\cdot,\cdot}$ | Kronecker delta |
| $\eta$ | measurement noise |
| $\theta$ | parameter vector |
| $v$ | process noise |
| $\sigma^2_{\Delta S}$ | stress signal variance |
| $\sigma^2_D$ | variance of the stress disturbance process |
| $\frac{dx}{dN}$ | fatigue crack growth rate |
| $f(\cdot)$ | evolution equation |
| $g(\cdot)$ | observation equation |
| $h(\cdot)$ | general fatigue crack growth rate model |
| $i$ | particle index |
| $N(\cdot,\cdot)$ | Gaussian probability density function |
| $N(0,\sigma^2_D)$ | stress disturbance process |
| $MVN(\cdot,\cdot)$ | multivariate Gaussian probability density function |
| $U([\cdot,\cdot])$ | uniform probability density function |
| $p(\cdot\vert\cdot)$ | conditioned probability density function |
| $\Gamma(\cdot)$ | Gamma function |
| $\widetilde{\Delta S}$ | equivalent stress range |

fatigue-induced damage propagation. However, none of them assume a variable load condition, typical of aeronautical, civil, and some mechanical structures. The first application of PF for variable-amplitude load conditions has been recently proposed in [16]. The authors used the Huang's model to mitigate the effect of the load ratio $R$ and the retardation effect due to overloads.[1] As indicated in the paper, the load condition is created by one load block with two different maximum load levels, and a well-defined number of cycles for each load level. The minimum load remains the same throughout the whole test, and the overstress (i.e. the value of the maximum load in the block) changes according to the experiments. Another recent tutorial on PF shows an identical application [17]. Even if the load condition is defined as a variable-amplitude fatigue, the load applied on the structure is deterministic, thereby the instantaneous value of the fatigue load in terms of mean and amplitude is known. Relevant works in the field of real-time prognosis and uncertainty in prognostics under variable-amplitude and random loading conditions has already been published by Sankararaman et al. [18,19]. These works underline the importance of a proper

[1] The mitigation of the retardation effect in the Huang's model is based on the model initially proposed by Wheeler in order to take the overload effects into account.