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Analysis of strength and wave velocity for micro-damaged elastic media

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ABSTRACT

A new macroscopic theoretical approach is proposed for strength analysis of damaged elastic bodies. Two types of defects are considered: cracks and round voids. The method applied is founded upon the Boundary Integral Equations (BIEs) method. The problem is studied for both anti-plane and in-plane mode of deformation. We give respective diagrams for calculated physical quantities. The results obtained in frames of the BIE method are compared with an alternative technique based on a propagation of ultrasonic impulses through the damaged media. There is evaluated a correlation between strength of the micro-damaged media and the wave speed of the of ultrasonic (US) impulse.

1. Introduction

It is well known in the engineering practice that elastic materials with arrays of micro-defects possess absolutely different mechanical properties when compared with homogeneous materials. Strength and fracture analysis of such elastic bodies with defects is an important problem in various fields of engineering science, in mechanics of concretes, composites, soils, ceramics, powder materials, etc. [1–4].

Various techniques are usually used to evaluate mechanical behavior of damaged materials. From the practical point of view, standard non-destructive techniques like ultrasonic evaluation may be applied to this problem [5,6]. However, application of such classical methods, efficient for metals, plastic materials, some kinds of composite materials, is faced with significant obstacles when operating with strongly damaged media. The nature of the principal difficulties in the application of ultrasonic techniques to strongly damaged media is connected with the fact that there is no reliable model, which can predict qualitative and quantitative correlation between the ultrasonic wave speed and the damage level (i.e. the number of flaws per unit volume) of the medium. In mechanics of concretes, there has been a lot of heated debate among researchers about the matter whether a change of the through-transmission wave velocity can correctly predict the change of mechanical properties of the material. In spite of sharp criticism [7], the evaluation of strength properties of concrete on the basis of the ultrasonic velocity in the through transmission technique is nowadays a conventional classical method of analysis.

Chronologically, first approaches to evaluate mechanical properties of micro-damaged media were founded on stochastic theories [8,9]. However, a more advanced approach has been developed based upon analytical, numerical and experimental investigation of the damaged media modeled as normal elastic bodies with a lot of randomly distributed defects [10]. Some

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Nomenclature

$\bar{u}$	displacement vector
$\overline{T}$	vector of the internal stress in the fractured medium
λ, μ	elastic moduli
n	unit normal to the elementary area inside the medium
$\bar{\tau}$	unit tangential vector on the elementary area inside the medium
τ_{ij}	stress tensor
x, y	Cartesian coordinates
ε	semi-length of the elementary crack
Φ	Green's function
U_{ij}	fundamental displacement tensor
Σ_{ij}	fundamental stress tensor
k_N	maximum stress intensity coefficient at the tip of a crack in the cracked medium with N cracks
T_s	mean time delay of Ultrasonic impulse
v_N	through-transmitted wave speed in the cracked medium

deterministic methods have also been applied by many authors to this subject of investigation [11–17]. Alternative methods are required to study systems of regularly distributed defects, like cracks in [18], where a good survey can be found too. Let us note that the methods proposed to both chaotic and regularly distributed arrays of flaws allowed the authors cited above to establish important qualitative and quantitative properties of the micro-damaged media. We only note in this connection that one of the most powerful approaches here is the application of the Boundary Integral Equations (BIEs) method, applied to both single [19] and multiple [10,14–16] defects.

In the present paper we further develop this promising direction of investigation, to apply it to the deterministic strength analysis of the damaged elastic solids loaded by a static uniform outer load. The problem is studied for both anti-plane and in-plane mode of deformation. The method is founded upon a numerical treatment of the system of BIEs holding over faces of all flaws forming a huge array. The defects are simulated as either cracks or round voids. In the case of cracked media the BIE itself **is allied to** a special treatment of the Boundary Element Methods (BEM) in the form of "displacement discontinuity method", where a square-root singularity of stress is not explicitly extracted from the structure of the solution but this is obtained numerically.

We give respective diagrams on results of the computations. The results obtained in frames of BIE are compared with an alternative method based on a through transmission technique with the use of ultrasonic impulses. There is evaluated a correlation between strength of the micro-damaged media and the through-transmission velocity of the ultrasonic (US) impulse.

It should also be noted that another approach to evaluate internal structure and related crack geometries is connected with the so-called "inverse" identification problems. Some our recent works with co-authors are devoted to this topic [20–22], where the reader can also find some other helpful references.

2. Basic equations of the elasticity theory for anti-plane and in-plane modes of deformation

The outer load τ_0 applied to the solid is modeled as a uniform stress distributed over a pair of the opposite straight lines $y = \pm h$ taken somewhere far from the domain containing the considered set of defects. In the case of the so-called "anti-plane" (or "shear-stress", SH) problem, this applied stress is directed perpendicularly to the considered plane (x, y), so that the mode of deformation is identical for all cross-sections $z = \text{const}$ [23]. Then in a fixed rectangular Cartesian coordinate system $Oxyz$ the non-trivial components of the stress tensor and the governing equation are

$$\tau_{xz}(x,y) = \mu \frac{\partial u_z(x,y)}{\partial x}, \quad \tau_{yz}(x,y) = \mu \frac{\partial u_z(x,y)}{\partial y}; \qquad \frac{\partial^2 u_z(x,y)}{\partial x^2} + \frac{\partial^2 u_z(x,y)}{\partial y^2} = 0, \tag{2.1}$$

where μ is the shear elastic modulus, and $\bar{u} = \{0, 0, u_z(x,y)\}$ is the displacement vector. The internal stress vector $\overline{T} = \{T_x, T_y, T_z\}$ on arbitrary elemental area with the unit normal $\bar{n} = \{n_x, n_y, 0\}$ in the considered case of anti-plane deformation has the only non-trivial component:

$$T_x = T_y = 0, \qquad T_z = \tau_{xz} n_x + \tau_{yz} n_y = \mu \left(\frac{\partial u_z}{\partial x} n_x + \frac{\partial u_z}{\partial y} n_y \right) = \mu \frac{\partial u_z}{\partial n}. \tag{2.2}$$

Then the boundary conditions over a set of cracks ℓ located in a certain domain inside the medium should be satisfied over the faces of all defects which are free of load. Let us represent the full solution as a sum of the stress caused by the applied load τ_0 in the undamaged medium and the perturbed one: