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The finite element discretized symplectic method for composite mode III cracks

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ABSTRACT

A finite element discretized symplectic method is proposed to compute the stress intensity factors (SIFs) of interface cracks in multi-material composites subjected to anti-plane loading. The whole body is divided into a near field containing a crack to be solved analytically and a far field to be solved by conventional finite elements. In the near field, analytical series solutions are found by the Hamiltonian formulation. The unknown displacements of the densely populated finite elements are transformed to a handful of unknown coefficients of the series. After combining with the far field, the SIFs are actually the first few coefficients.

1. Introduction

With the development of high-tech industries such as aircrafts, vehicles and ships, homogeneous materials are unable to meet performance and reliability requirements for modern engineering applications. Composites are often used to satisfy these requirements. However, it is found that, for composites, the interface is easy to debond and fracture due to singular stresses induced by the sudden change of material properties along interfaces. The presence of delamination cracks alters the integrity of the structure and deteriorates the functional requirements on mechanical, thermal and electric properties, etc. Therefore, it is of great importance to study the interface cracks in composites or composite structures.

The evaluation of interface cracks is a problem of long standing in fracture mechanics. For the anti-plane problems, the pioneering work by Erdogan and associates [1–3] on anti-plane shear offered numerous solutions for two-phase solids weakened by several single or multiple crack configurations. After that, much research has been devoted to the analysis of anti-plane interfacial fracture problems. Wu and Chiu [4] analyzed anti-plane shear interface cracks in a finite anisotropic composite body and found the lower bound for stress intensity factor (SIF) was in the case of isotropic bimaterial. Sze and Wang [5] proposed a simple finite element formulation for computing stress singularities at bimaterial interfaces. Lee and Earmme [6] analyzed an interfacial edge crack of a length a in two bonded dissimilar anisotropic quarter planes having an anti-plane singularity and derived the expression of SIFs. Dvorak and Sunorov [7] evaluated SIFs of planar mode III shear cracks perpendicular to a nearby interface between two isotropic elastic solids based on superposition of singular near tip

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Nomenclature

$\mathbf{c}$	coefficient vector of the symplectic series
E, ν, G	elastic modulus, Poisson's ratio and shear modulus
$\mathbf{f}$	global nodal force vector
$\mathbf{f}_N$	nodal force vector in the near field
$\mathbf{f}_F$	nodal force vector in the far field
$\mathbf{H}$	Hamiltonian operator matrix
K_3	mode III stress intensity factor
k_3	normalized mode III stress intensity factor
$\mathbf{K}$	global stiffness matrix
N_N, N_F	node number in near field and far field
N_S	number of symplectic series
(r, θ)	polar coordinate system
$\mathbf{u}$	global nodal displacement vector
$\mathbf{u}_N$	nodal displacement vector in the near field
$\mathbf{u}_F$	nodal displacement vector in the far field
w	z-axis displacement
(x, y)	Cartesian coordinate system
τ_{ij}	components of stresses
μ, $\boldsymbol{\psi}$	symplectic eigenvalue and eigenfunction
$\boldsymbol{\Psi}$	total unknown vector in the Hamiltonian system

stress and displacement fields generated by both the main crack and certain image cracks. Li et al. [8,9] performed the anti-plane transient fracture analysis of the bonded functionally graded material (FGM) half-planes and strips, and discussed the enhancement of the capacity of FGMs interface. Ding and Li [10] analyzed the interface cracking between a FGM and an elastic substrate under anti-plane shear loads. The influencing factors on the values of SIFs were discussed. Matbuly and Nassar [11,12] considered the mode III crack perpendicular to the interface of a bi-strip composite and double interface cracks located between two dissimilar orthotropic plates and obtained the expressions of SIFs. Shahani et al. [13–16] studied the interfacial fracture behaviors in various bonded wedges. The singularities at the apex of wedges and SIFs for the interface cracks were investigated. Leung and Tsang [17] employed the fractal finite element method (FFEM) to determine mode III SIFs. In the same manner, Treifi et al. [18,19] extended the FFEM to compute the SIFs of bimaterial cracked/notched bodies subjected to anti-plane shear loading. Chue et al. [20–23] studied the mode III crack and interface crack problems of anisotropic wedges, junctions and bonded functionally graded strips and derived the formulae of mode III SIFs. Chen et al. [24–26] worked on the composite finite wedges and obtained the SIFs for interface cracks not only along the interface but also terminating at the interface. Beom and Jang [27,28] obtained the SIFs for the inner cracks and interface cracks in an anisotropic bimaterial wedge based on a linear transformation method. Yao et al. [29–31] developed a novel singular finite element and investigated the stress singularity on multi-material wedges under anti-plane deformation.

In view of the available literatures, it is found that all of works concerned with the SIF, showing that it is an important fracture parameter in failure design studies. Fast, reliable and accurate computations of SIFs are necessary in the design of new structures or in the assessment of the integrity of existing structures. However, most of the theoretical works on the interfacial fracture analysis were limited by the geometric boundary conditions and locations of interface cracks. They were concentrated on the infinite/half-infinite media or the finite media having simple geometries, and the interface cracks were usually assumed to be along the interface.

In engineering practice, the composites or composite structures in service always have finite size and complex geometries. In addition, the interface cracks may exist in two cases: the crack lying along the interface and the crack terminating at the interface. Although numerical methods are capable of any geometric boundary conditions and crack cases, no analytical solutions can be obtained in the calculations. Large amount of data cannot provide direct assistance to the designers. Therefore there is still a need for carrying out a systematic study on the interface cracks under anti-plane loading.

The finite element discretized symplectic method (FEDSM) [32,33] has been proved to be an efficient and accurate method for fracture analysis. In this paper, the anti-plane stress and displacement expressions around the interface crack tip are derived analytically by a symplectic approach and employed as global functions to extend the FEDSM to compute the mode III SIFs for the interface cracks in composite structures. The symplectic approach for elasticity was first presented by Zhong and collaborators [34] and has been rapidly extended in a number of research fields, including theory of thin plates [35–37], viscoelasticity [38], magneto-electro-elasticity [39], fluid mechanics [40], fracture mechanics [41,42], etc. For further details on the theory and applications of the symplectic approach, the reader can refer to the review article by Lim and Xu [43]. The FEDSM brings together the agility of the FEM and the accuracy of the analytical symplectic method. The overall body is separated into a finite size singular stress region near the crack tip and a regular region far away from the crack tip, i.e., near field and far field. Both the near and far fields are meshed by any suitable conventional elements.