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Instability of a planar liquid sheet with surrounding fluids between two parallel walls

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Abstract

We investigate the linear and nonlinear instability of a planar liquid sheet with surrounding fluids between two parallel plane solid walls. Linear analysis shows that the maximum temporal growth rate and unstable wave number region of disturbances increase for the dilational and sinuous modes when the gap between the sheet and the wall decreases. The walls have more influence on the instability when the density ratio of the surrounding fluid to the sheet and/or the Weber number decrease. On the other hand, nonlinear analysis is performed by means of the discrete vortex method, where double vortex rows and their mirror images are placed so as to satisfy the boundary condition on the walls. Numerical results show that the walls enhance nonlinearity, which causes deformation and distortion of the sheet, whereas the nonlinearity diminishes linear growth rates except for long dilational disturbances. In particular, as the walls are placed more closely to the sheet, local sheet thinning becomes more pronounced in the long dilational mode, while the dilational mode is more strongly induced from the sinuous mode through monotonic or periodic energy exchanges between the two modes.

Keywords: Instability; Nonlinear; Liquid sheet; Surface tension; Discrete vortex method; Solid wall

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1. Introduction

It is known that instability of a planar liquid sheet is of great importance not only in typical problems of fluid dynamics, but also in applications to industrial and engineering problems such as atomization, coating techniques and polymer processing (Lefebvre, 1989; Middleman, 1995; Ramos, 1988). For disturbances on a planar sheet, the sheet is destabilized by the shearing motion with surrounding fluids, which is called Kelvin–Helmholtz instability, while the sheet is stabilized by the surface tension on sheet surfaces. As a resultant of these two effects, the sheet becomes unstable only for long wave disturbances. Then, the sheet with finite thickness permits two distinct surface modes of infinitesimal disturbances, that is, the dilational and sinuous modes, as shown in Fig. 1. In the dilational mode, both sheet surfaces are out of phase and symmetric with respect to the sheet mid-plane, while in the sinuous mode they are in phase and the mid-plane is flexible.

Linear temporal instability of a planar sheet without viscosity has been shown by several investigators (Squire, 1953; Hagerty and Shea, 1955; Taylor, 1959; Rangel and Sirignano, 1991). Introducing the density ratio ρ of the surrounding fluid to the sheet and the Weber number Wb defined as the ratio of inertia to surface tension, their results are summarized as follows: the dilational mode always has a larger region of unstable wave number than the sinuous mode. However, the sinuous mode has a larger growth rate than the dilational mode if ρ is small and/or Wb is large. If $\rho \neq 0$, the sinuous mode is unstable for Wb larger than a critical value, while the dilational mode is unstable for all Wb.

Since instability may bring about large deformation and distortion of the sheet even if the surface disturbance is initially infinitesimal, nonlinearity becomes important. For quantitative understanding of this, nonlinear analysis has been performed analytically and numerically. It is shown that a long wave approximation is effective to analytically derive nonlinear evolution equations for large deformation of the sheet without surroundings (Mehring and Sirignano, 1999; Yoshinaga and Kotani, 2001; Yoshinaga and Kan, 2006). On the other hand, breakup or disintegration of the sheet can be numerically examined without using such an approximation, although only a few numerical methods are available for problems of free boundaries with surface tension (Peyret and Taylor, 1983). Recently, Rangel and Sirignano (1991) numerically examined large deformation of a planar sheet with surrounding fluids by using the discrete vortex method under spatial periodicity. They showed that nonlinear growth rates of the two modes are always less than the linear ones. Then, the sheet may break up at every one-wavelength of a dilational mode disturbance and at every half-wavelength of a sinuous mode disturbance.

In the above analysis, the surrounding fluids are assumed to be externally unbounded above and below the sheet surfaces. In practice, however, the surroundings are often bounded externally by solid walls or others. In spite of this, the nonlinear behavior of such a sheet with external boundaries has not been examined, though the linear instability in this case has been examined for a semi-infinite and viscous sheet (Lin, 2003).

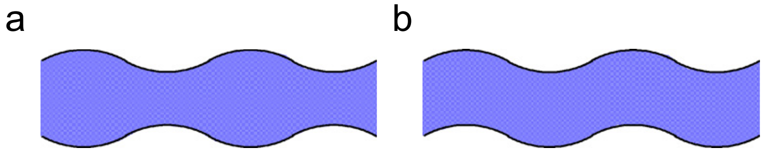

Fig. 1. Schematic of the two surface modes on a planar liquid sheet: (a) dilational mode and (b) sinuous mode.