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Static and free vibration analyses of small-scale functionally graded beams possessing a variable length scale parameter using different beam theories

Reza Aghazadeh, Ender Cigeroglu, Serkan Dag*

Department of Mechanical Engineering, Middle East Technical University, Ankara 06800, Turkey

A R T I C L E I N F O

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A B S T R A C T

This article puts forward a modified couple stress theory based approach of analysis for small-scale functionally graded beams, that possess a variable length scale parameter. Presented procedures are capable of predicting static and dynamic beam responses according to three different beam theories, namely: Euler–Bernoulli beam theory, Timoshenko beam theory and third-order shear deformation beam theory. A variational method is used in conjunction with the modified couple stress theory to derive the governing partial differential equations. All properties of the small-scale functionally graded beams – including the length scale parameter – are assumed to be functions of the thickness coordinate in the derivations. The governing equations are solved numerically through the use of the differential quadrature method (DQM). Numerical results are generated for small-scale functionally graded beams, that comprise ceramic and metallic materials as constituent phases. Both small-scale beams subjected to static loading and those undergoing free vibrations are considered in the computations. Comparisons of the numerical results to those available in the literature point out that developed techniques lead to results of high accuracy. Further numerical results are provided, which demonstrate the responses of small-scale functionally graded beams estimated by the three different beam theories as well as provide insight into the influences of material parameters upon the static deflections and natural vibration frequencies.

1. Introduction

Micro-and nano-scale components are currently being used in technological applications such as microelectromechanical systems, nanoelectromechanical systems, atomic force microscopes, and biosensors (Coutu Jr. et al., 2004; Mahdavi et al., 2008; Pei et al., 2004; Younis et al., 2006). These components, which possess dimensions at the order of micrometers or nanometers, are generally collectively referred to as small-scale structures. Such structural elements exhibit size-dependent behavior, which cannot be predicted by the conventional continuum theories. For example, the experiments conducted by Lam et al. (2003) on micro-cantilever beams indicate that, tip deflection of a micro-cantilever beam varies with beam thickness even when the ratio of the beam length to the beam thickness is kept constant. This observation contradicts the prediction of the classical elasticity theory, which anticipates a tip deflection independent of the beam thickness as long as the beam length to beam thickness ratio is fixed. Experimental results reported by Andrew and Jonathan (2005) clearly illustrate that stiffnesses of micro-cantilever plates are at least four times greater than those calculated by the classical theory.

A continuum theory incorporating the influence of the intrinsic microstructural length scale into the analysis needs to be employed to be able to examine the behavior of small-scale structures. As examples of such continuum theories, we can mention nonlocal elasticity (Eringen, 1972), surface elasticity (Gurtin et al., 1998), strain gradient theories (Aifantis, 1999; Fleck and Hutchinson, 1997, 2001; Lam et al., 2003) and couple stress theories (Mindlin and Tiersten, 1962; Toupin, 1962; Yang et al., 2002). The strain gradient theory proposed by Lam et al. (2003), and the modified couple stress theory put forward by Yang et al. (2002) are commonly used in structural analysis of small-scale components. The strain gradient theory of Lam et al. (2003) is developed in terms of three material length scale parameters, which are used to relate dilatation gradient, deviatoric stretch gradient, and symmetric part of the rotational gradient to the relevant stress components. In the

* Corresponding author. Tel.: +90 312 2102580; fax: +90 312 2102536.
E-mail address: sdag@metu.edu.tr (S. Dag).

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modified couple stress theory of Yang et al. (2002), the effects of the dilatation gradient and the deviatoric stretch gradient are neglected, and only a single material length scale parameter is required, which relates the couple stress to the symmetric part of the rotational gradient.

Modified couple stress theory and strain gradient theory are used in various studies to address structural mechanics problems involving homogeneous materials (Asghari et al., 2010a; Ma et al., 2008, 2010) and advanced composites such as functionally graded materials (FGMs). Graded materials belong to a certain class of multiphase composites, which possess smooth spatial variations in the volume fractions of the constituents. Due to these intentionally introduced variations, the microstructure is inhomogeneous and material properties are functions of the spatial coordinates. Asghari et al. (2010b), Asghari et al. (2011), and Ke and Wang (2011) utilized modified couple stress theory to solve structural problems regarding small-scale FGM beams. Ansari et al. (2011) employed strain gradient approach and Timoshenko beam theory to analyze small-scale graded beams undergoing free vibrations. Reddy (2011) and Ke at al. (2012) incorporated von Karman geometric nonlinearity into the modified couple stress theory based formulation in the analysis of small-scale functionally graded beams.

In all studies on small-scale functionally graded beams mentioned in the previous paragraph, the length scale parameters used in the formulation are taken as constants. Note that a single length scale parameter is needed in modified couple stress theory whereas strain gradient elasticity requires the use of three different length scale parameters. Length scale parameter is essentially an elastic property within the contexts of both theories. For example, in modified couple stress theory, the ratio of modulus of curvature to shear modulus is defined as the square of the length scale parameter (Mindlin, 1963; Park and Gao, 2006). Both modulus of curvature and shear modulus are elastic properties implying that the length scale parameter is itself an elastic property. As a consequence, for a functionally graded medium, the length scale parameter is also in general a function of the spatial coordinates similar to the other elastic properties. Thus, a general formulation should take into account the variation in the length scale parameter as well. The only study in the literature that considers the variations in the length scale parameters seems to be that by Kahrobaiyan et al. (2012). In this article, the authors present analysis methods based on the use of strain gradient elasticity in conjunction with the Euler–Bernoulli beam theory. However, Euler–Bernoulli theory is built on certain restrictive assumptions such as the assumption of zero shear strain; and in a number of problems this theory has a tendency to overestimate the normal stresses in small-scale beams.

The main objective of the present study is to set forth a more general method to examine the response of small-scale functionally graded beams possessing a variable length scale parameter. The methodology is based on the modified couple stress theory; and the formulation is carried out in such a way that, proposed techniques make it possible to predict the beam response by means of three different beam theories, which are: Euler–Bernoulli beam theory, Timoshenko beam theory, and third-order beam theory. Both, beams subjected to static loading and those undergoing free vibrations are considered in the developments. Governing partial differential equations are derived by following the variational approach and using Hamilton's principle. All material properties of the small-scale functionally graded beams – including the length scale parameter – are assumed to be functions of the thickness coordinate in the derivations. Governing equations are solved numerically by employing the differential quadrature method (DQM). Comparisons of the numerical results to those available in the technical literature indicate that a high level of accuracy is achieved by the application of the proposed methods. Further numerical results are presented, which include the predictions of the static and dynamic beam responses according to Euler–Bernoulli, Timoshenko, and third-order beam theories as well as the results demonstrating the influences of material parameters upon quantities such as static deflection and natural vibration frequencies.

The outline of the paper is as follows: In Section 2, we delineate the theoretical preliminaries regarding modified couple stress theory and the beam theories considered. Section 3 details the derivation procedures of the governing partial differential equations. Numerical solution technique and the differential quadrature method are elucidated in Section 4. Results and parametric analyses are presented in Section 5. The paper concludes with Section 6, where we provide our final remarks.

2. Theoretical preliminaries

The geometry and deformed shape of the small-scale functionally graded beam examined in this study are shown in Fig. 1. The thickness and the length are respectively symbolized by h and L. The cross-section is rectangular and the width is denoted by b. Material properties vary continuously across the thickness direction; and the beam is 100% metal at the lower surface $x_3 = -h/2$ and 100% ceramic at $x_3 = h/2$. Moreover, the structure depicted is assumed to be either statically loaded or undergoing free vibrations.

According to the modified couple stress theory, total strain energy of a loaded beam is given as follows (Yang et al., 2002):

$$U = \frac{1}{2}\int_V \left(\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij}\right)\mathrm{d}V, \qquad (1)$$

where V is volume, σ_{ij} designates the Cauchy stress tensor, ε_{ij} is the strain tensor, m_{ij} stands for the deviatoric part of the couple stress tensor, and χ_{ij} represents the symmetric curvature tensor. The tensors ε_{ij} and χ_{ij} are defined by

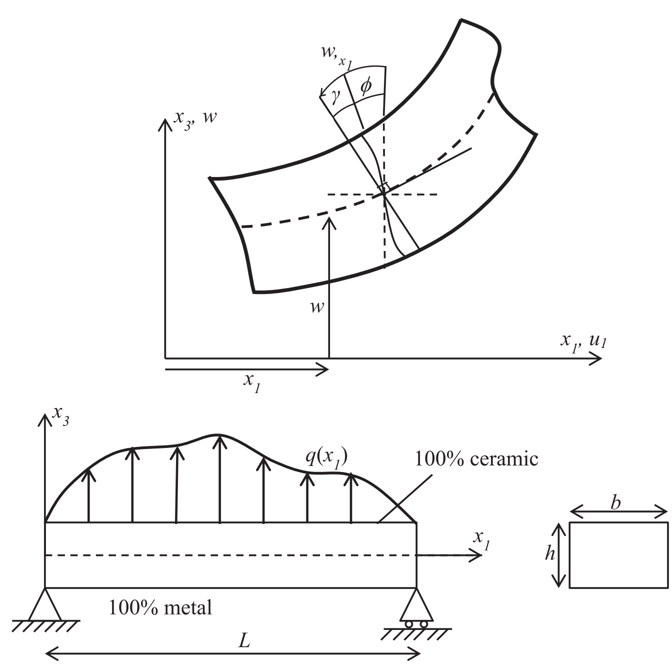

Fig. 1. Geometry of the small-scale functionally graded beam and the deformed shape.