European Journal of Mechanics A/Solids 46 (2014) 42–53

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

Nonlinear dynamic analysis of eccentrically stiffened functionally graded circular cylindrical thin shells under external pressure and surrounded by an elastic medium

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A R T I C L E I N F O

Article history:
Received 17 July 2013
Accepted 9 February 2014
Available online 18 February 2014

Keywords:
Functionally graded material
Nonlinear dynamic analysis
Stiffened circular cylindrical shell

A B S T R A C T

A semi-analytical approach eccentrically stiffened functionally graded circular cylindrical shells surrounded by an elastic medium subjected to external pressure is presented The elastic medium is assumed as two-parameter elastic foundation model proposed by Pasternak. Based on the classical thin shell theory with the geometrical nonlinearity in von Karman–Donnell sense, the smeared stiffeners technique and Galerkin method, this paper deals the nonlinear dynamic problem. The approximate three-term solution of deflection shape is chosen and the frequency–amplitude relation of nonlinear vibration is obtained in explicit form. The nonlinear dynamic responses are analyzed by using fourth order Runge–Kutta method and the nonlinear dynamic buckling behavior of stiffened functionally graded shells is investigated according to Budiansky–Roth criterion. Results are given to evaluate effects of stiffener, elastic foundation and input factors on the frequency–amplitude curves, natural frequencies, nonlinear responses and nonlinear dynamic buckling loads of functionally graded cylindrical shells.

1. Introduction

Functionally graded material (FGM) cylindrical shell has become popular in engineering designs of coating of nuclear reactors and space shuttle. The static and dynamic behavior of FGM cylindrical shell attracts special attention of a lot of researchers in the world.

In static analysis of FGM cylindrical shells, many studies have been focused on the buckling and postbuckling of shells under mechanic and thermal loading. Shen (2003) presented the nonlinear postbuckling of perfect and imperfect FGM cylindrical thin shells in thermal environments under lateral pressure by using the classical shell theory with the geometrical nonlinearity in von Karman–Donnell sense. By using higher order shear deformation theory; this author (Shen, 2005) continued to investigate the postbuckling of FGM hybrid cylindrical shells in thermal environments under axial loading. Huang and Han (2008, 2009a, 2009b, 2010a, 2010b) studied the buckling and postbuckling of un-stiffened FGM cylindrical shells under torsion load, axial compression, radial pressure, combined axial compression and radial pressure based on the Donnell shell theory and the nonlinear strain–displacement relations of large deformation. Shen (2009b) investigated the torsional buckling and postbuckling of FGM cylindrical shells in thermal environments. The non-linear static buckling of FGM conical shells which is more general than cylindrical shells, were studied by Sofiyev (2011a,b). Zozulya and Zhang (2012) studied the behavior of functionally graded axisymmetric cylindrical shells based on the high order theory.

For dynamic analysis of FGM cylindrical shells, Darabi et al. (2008) presented respectively linear and nonlinear parametric resonance analyses for un-stiffened FGM cylindrical shells. Sofiyev and Schnack (2004) and Sofiyev (2005) obtained critical parameters for un-stiffened cylindrical thin shells under linearly increasing dynamic torsional loading and under a periodic axial impulsive loading by using the Galerkin technique together with Ritz type variation method. Sheng and Wang (2008) presented the thermo-mechanical vibration analysis of FGM shell with flowing fluid. Sofiyev (2003, 2004, 2009, 2012) and Deniz and Sofiyev (2013) were investigated the vibration and dynamic instability of FGM conical shells. Hong (2013) studied thermal vibration of magnetostrictive FGM cylindrical shells. Huang and Han (2010c) presented the nonlinear dynamic buckling problems of un-stiffened functionally graded cylindrical shells subjected to time-dependent axial

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http://dx.doi.org/10.1016/j.euromechsol.2014.02.008
0997-7538/

load by using the Budiansky–Roth dynamic buckling criterion (Budiansky and Roth, 1962). Various effects of the inhomogeneous parameter, loading speed, dimension parameters; environmental temperature rise and initial geometrical imperfection on nonlinear dynamic buckling were discussed.

For FGM cylindrical shell surrounded by an elastic foundation, the postbuckling of shear deformable FGM cylindrical shells surrounded by an elastic medium was studied by Shen (2009a). Shen et al. (2010) investigated postbuckling of internal pressure loaded FGM cylindrical shells surrounded by an elastic medium. Bagherizadeh et al. (2011) investigated mechanical buckling of FGM cylindrical shells surrounded by Pasternak elastic foundation. Sofiyev (2010) analyzed the buckling of FGM circular shells under combined loads and resting on the Pasternak type elastic foundation. Torsional vibration and stability of functionally graded orthotropic cylindrical shells on elastic foundations is presented by Najafov et al. (2013). For the FGM conical shell – general case of FGM cylindrical shells, mechanic behavior of shell on elastic foundation was studied by Sofiyev (2011c), Najafov and Sofiyev (2013), Sofiyev and Kuruoglu (2013).

In practice, FGM plates and shells, as other composite structures, usually reinforced by stiffeners system to provide the benefit of added load carrying capability with a relatively small additional weight. Thus study on nonlinear static and dynamic behavior of theses structures are significant practical problem. However, up to date, the investigation on this field has received comparatively little attention. Recently, Najafizadeh et al. (2009) have studied linear static buckling of FGM axially loaded cylindrical shell reinforced by ring and stringer FGM stiffeners. Bich et al. (2011, 2012, 2013) have investigated the nonlinear static and dynamic analysis of FGM plates, cylindrical panels and shallow shells with eccentrically homogeneous stiffener system. Dung and Hoa (2013a, 2013b) presented an analytical study of nonlinear static buckling and post-buckling analysis of eccentrically stiffened functionally graded circular cylindrical shells under external pressure and torsional load with FGM stiffeners and approximate three-term solution of deflection taking into account the nonlinear buckling shape.

The review of the literature signifies that there are very little researches on the nonlinear dynamic analysis of FGM stiffened shells surrounded by an elastic foundation by analytical approach. In this paper, the dynamic behavior of eccentrically stiffened FGM (ES-FGM) cylindrical circular shells reinforced by eccentrically ring and stringer stiffener system on internal and (or) external surface of shell under external pressure loads is investigated. The nonlinear dynamic equations are derived by using the classical shell theory with the nonlinear strain–displacement relation of large deflection, the smeared stiffeners technique and Galerkin method. The present novelty is that an approximate three-term solution of deflection including the pre-buckling shape, the linear buckling shape and the nonlinear buckling shape are more correctly chosen and the frequency–amplitude relation of nonlinear vibration is obtained in explicit form. In addition, the nonlinear dynamic responses are found by using fourth order Runge–Kutta method and the dynamic buckling loads of stiffened FGM shells are investigated according to Budiansky–Roth criterion. The results show that the stiffener, volume-fractions index and geometrical parameters strongly influence to the dynamic behavior of shells.

2. Formulation

2.1. FGM power law properties

Functionally graded material in this paper, is assumed to be made from a mixture of ceramic and metal in two cases: inside ceramic surface, outside metal surface and outside ceramic surface, inside metal surface. The volume-fractions is assumed to be given by a power law

$$V_{in} = V_{in}(z) = \left(\frac{2z+h}{2h}\right)^k, \quad V_{ou} = V_{ou}(z) = 1 - V_{in}(z), \tag{1}$$

where h is the thickness of shell; $k \geq 0$ is the volume-fraction index; z is the thickness coordinate and varies from $-h/2$ to $h/2$; the subscripts *in* and *ou* refer to the inside and outside material constituents, respectively.

For case of inside ceramic surface and outside metal surface $V_{in} = V_c$ and $V_{ou} = V_m$, for the case of outside ceramic surface and inside metal surface $V_{in} = V_m$ and $V_{ou} = V_c$. In which, V_c is volume-fraction of ceramic and V_m is volume-fraction of metal.

Effective properties Pr_{eff} of FGM shell are determined by linear rule of mixture as

$$Pr_{eff} = Pr_{ou}(z)V_{ou}(z) + Pr_{in}(z)V_{in}(z). \tag{2}$$

According to the mentioned law, the Young's modulus and the mass density of shell can be expressed in the form

$$\begin{aligned} E(z) &= E_{ou}V_{ou} + E_{in}V_{in} = E_{ou} + (E_{in} - E_{ou})\left(\frac{2z+h}{2h}\right)^k, \\ \rho(z) &= \rho_{ou}V_{ou} + \rho_{in}V_{in} = \rho_{ou} + (\rho_{in} - \rho_{ou})\left(\frac{2z+h}{2h}\right)^k, \end{aligned} \tag{3}$$

For case of inside ceramic surface and outside metal surface $E_{in} = E_c$, $\rho_{in} = \rho_c$ and $E_{ou} = E_m$, $\rho_{ou} = \rho_m$, for the case of outside ceramic surface and inside metal surface $E_{in} = E_m$, $\rho_{in} = \rho_m$ and $E_{ou} = E_c$, $\rho_{ou} = \rho_c$. E_c, ρ_c, E_m, ρ_m are the Young's modulus and the mass density of ceramic and metal, respectively.

2.2. Constitutive relations and governing equations

Consider a functionally graded cylindrical thin shell surrounded by an elastic foundation with length L, mean radius R and reinforced by closely spaced (Najafizadeh et al., 2009; Brush and Almroth, 1975; Reddy and Starnes, 1993) pure-metal ring and stringer stiffener systems (see Fig. 1). The stiffener is located at outside surface for outside metal surface case and at inside surface for inside metal surface case. The origin of the coordinate O locates on the middle surface and at the left end of the shell, x,$y = R\theta$ and z axes are in the axial, circumferential, and inward radial directions respectively.

According to the von Karman nonlinear strain–displacement relations (Brush and Almroth, 1975), the strain components at the middle surface of perfect circular cylindrical shells are the form

$$\begin{aligned} \varepsilon_x^0 &= \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \\ \varepsilon_y^0 &= \frac{\partial v}{\partial y} - \frac{w}{R} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2, \\ \gamma_{xy}^0 &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}, \\ \chi_x &= \frac{\partial^2 w}{\partial x^2}, \quad \chi_y = \frac{\partial^2 w}{\partial y^2}, \quad \chi_{xy} = \frac{\partial^2 w}{\partial x \partial y}, \end{aligned} \tag{4}$$

where ε_x^0 and ε_y^0 are normal strains, γ_{xy}^0 is the shear strain at the middle surface of shell, χ_x, χ_y, χ_{xy} are the change of curvatures and twist of shell, and $u = u(x,y)$, $v = v(x,y)$, $w = w(x,y)$ are displacements along x, y and z axes respectively.