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On thermodynamic functions in thermoelasticity without energy dissipation

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A R T I C L E I N F O

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A B S T R A C T

Using a systematic procedure based on convex/concave functions and Legendre transforms, the consistent set of the thermodynamic functions in the framework of the Green and Naghdi (GN) thermoelasticity without energy dissipation is addressed. Starting from the free energy of GN, the thermodynamic potentials, i.e. internal energy, enthalpy and Gibbs free energy, are derived. Moreover it is shown that four more thermodynamic potentials, named alternative, can be provided. The relations between these functions are provided and the constitutive relations are obtained extending the general methodology used for the classical thermoelastic behaviour.

1. Introduction

In many engineering applications, non-isothermal elastic problems are of great interest and have become increasingly important. As well-known the classical theory of thermoelasticity, see e.g. Biot (1956), implies that the heat conduction occurs at an infinite speed so that some behaviours, such as heat wave propagation phenomena, cannot be successfully captured. As a consequence a large amount of papers are devoted to the analysis of generalized theories of thermoelasticity.

The first theory of this type was developed by Lord and Shulman (1967) who considered anisotropic elastic theory with one relaxation time parameter into the Fourier heat conduction equation. As a result the related heat equation turns out to be of a hyperbolic type. Such a theory has been later extended by Dhaliwal and Sherief (1980) to the case of anisotropic media. A further generalization of the Lord and Shulman theory was proposed by Green and Lindsay (1972) by considering a thermoelastic model with two relaxation time parameters.

A new thermoelastic theory without energy dissipation has been proposed by Green and Naghdi (1991, 1993) in which the internal rate of production of entropy is identically zero. This thermoelastic theory introduces the so-called thermal displacement, which is related to the usual temperature by a differential relation, and uses a general entropy balance as postulated in Green and Naghdi (1977). Such a theory incorporates the approach based on Fourier's law (type I), a theory which allows for heat transmission at finite speed and for which there is no energy dissipation (type II) and a model which allows finite wave propagation as well as energy dissipation (type III). Uniqueness theorems in the case of the linearized version of this theory have been given by Green and Naghdi (1993), Chandrasekharaiah (1996, 1998) and Quintanilla (2002).

Further theoretical aspects of the linear Green and Naghdi (GN) theory of thermoelasticity without energy dissipation was studied by Chandrasekharaiah (1996), Iesan (1998), Quintanilla and Straughan (2000), Kalpakides and Maugin (2004).

The approach of GN has been applied to the thermomechanically coupled problem for homogeneous and isotropic materials in Bargmann and Steinmann (2006), for bodies with microstructure and microtemperature in Iesan and Quintanilla (2009). Further application of the GN theory of type II can be found in Hosseini and Abolbashari (2012), where heat wave propagation in functionally graded thick hollow cylinder is analysed, Chiriță and Ciarletta (2010a), where non-standard conditions are imposed for the linear analysis of a prismatic cylinder made by a thermoelastic homogeneous and anisotropic material and Yu et al. (2012) for the analysis of the thermoelastic waves propagation in orthotropic functionally graded plate. Variational aspects of the GN theory are treated in Bargmann and Steinmann (2006, 2008), Chiriță and Ciarletta (2010b) for the analysis of thermoelastic inhomogeneous and anisotropic materials, Youssef (2011) where a uniqueness theorem is proved for two-temperature generalized thermoelasticity

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and a general variational formulation in a nonlocal context is presented in Marotti de Sciarra (2009c).

An account of theories of heat conduction where the temperature travels as a wave with a finite speed is reported in Straughan (2011). The increase of engineering applications at the nano- and micro-scale requires the development of accuracy models to predict their responses, see e.g. Barretta and Marotti de Sciarra (2013), Barretta et al. (2013), and studies on non-classical diffusion are of outmost importance since there is evidence that thermal motion at the micro-scale is via a wave mechanism as opposed to by diffusion (Straughan, 2011).

Nonlinear GN theory of type II is studied by Quintanilla and Sivaloganathan (2012) where various analyses on GN model, including the existence and uniqueness of quasistatic solutions and the possibility of thermally activated instability, are considered.

A theoretical aspect for the GN theory without dissipation (type II) that deserves further studies is the systematic and unified definition of thermodynamic potentials and of their alternative formulations for the analysis of the constitutive behaviour of a thermoelastic material.

In the framework of coupled nonlinear elastic thermomechanical problems, the present paper has the objective to establish a general methodology to derive the energy functions which characterize the nonlinear thermoelastic model without dissipation. Further the relations between the thermodynamic functions and their alternative formulations are obtained.

A consistent set of the eight thermodynamic functions has been derived into the framework of convex analysis and conjugate functions. In particular it is obtained the four specific thermodynamic potentials, i.e. internal energy, Helmholtz free energy, enthalpy and Gibbs free energy, and their four alternative forms.

In the considered GN model, the thermodynamic functions depend on three state variables where the dual set of constitutive state variables are given by strain, stresses, temperature, entropy, gradient of thermal displacement and entropy flux.

Using a systematic procedure based on Legendre transforms, the thermodynamic potentials are expressed in terms of different combinations of the abovementioned state variables obtaining a set of eight alternative functions. A characteristic feature of the proposed approach is that the derivatives of the thermodynamic potentials and of their alternative forms provide different expressions of the constitutive relations which are all equivalent each other.

We then provide the generalizations to the GN thermoelastic framework of the classical Legendre transform relating elastic and complementary energies.

Then a set of relations involving the thermoelastic work has been straightforwardly specialized in an example considering a one-dimensional loading path (isoentropic, isothermal, constant stress and constant strain). Hence the link with the classical thermoelastic relations is also shown by comparing the new relations with the classical ones reported in Lubarda (2004).

The layout of this paper is as follows. In Section 2 the main issues concerning the GN thermoelastic theory are provided. Then evaluating the conjugate of the free energy with respect to one, two and three (the complete set) state variables, the thermodynamic functions and their alternative formulations are obtained. In Section 3 the Legendre transform between the thermodynamic and alternative functions are given by appealing to the general rules of convex/concave functions and the constitutive relations in terms of the thermodynamic functions and of their alternative formulations are obtained. In Section 4 the general expressions of the thermodynamic potentials for the GN nonlinear model are then specialized to the linear coupled isotropic thermoelastic behaviour. In Section 5 a one-dimensional example illustrates the relations between the thermodynamic potentials and the thermal work. The connections with the existing expressions in literature for the classical linear coupled thermoelasticity are also provided.

Further development of this issue will concern the thermodynamic functions in coupled thermomechanical problems with damage, particularly if damage is due to both mechanical and thermal strains and if the material is exposed to elevated temperatures (see e.g. Stabler and Baker, 2000), and related nonlocal problems which have to be considered to avoid localization problems (see for example Marotti de Sciarra, 2009a, 2012).

2. Energy functions

The classical thermodynamics of fluids makes use of the quantities pressure, temperature, specific volume and specific entropy. The specific quantities are defined per unit mass. Moreover, four energy functions are defined which are the specific internal energy U, the specific Helmholtz free energy Φ, the specific enthalpy H and the specific Gibbs free energy G. The four energies are related by a series of Legendre transformations (e.g. Callen, 1985).

In applying thermodynamics to solids, pressure and specific volume are replaced by stress and strain tensors. In small strain analysis if the specific quantities are defined per unit volume rather than per unit mass, the initial density appears as a multiplicative factor throughout the analysis so that it can be dropped. Thus the four energy functions in thermodynamics of solids assume a slightly different meaning from those in classical thermodynamics (see e.g. Houlsby and Puzrin, 2006).

The starting point of thermodynamics is the hypothesis that at any instant of a thermomechanical process, the thermodynamic state at a given point can be completely determined by the knowledge of a finite number of state variables. The thermodynamic state depends only on the instantaneous value of the state variables and not on their past history.

In this paper we analyse the Green and Naghdi (GN) thermoelastic framework of type II for local constitutive elastic models using the properties of conjugate functions.

The strain tensor is denoted by $\boldsymbol{\varepsilon}$, the dual state variable is the stress tensor $\boldsymbol{\sigma}$. The scalar product between dual quantities (simple or double index saturation operation between vectors or tensors) has the mechanical meaning of the internal virtual work, and is denoted by the symbol *.

The GN model introduces a scalar variable α, called the thermal displacement, which is related to the temperature θ by the relation:

$$\alpha(x,t) = \int_0^t \vartheta(x,\tau)d\tau + \alpha_0(x,0) \tag{1}$$

where $\mathbf{x}$ is a point pertaining to the thermoelastic body defined on a regular bounded domain Ω of an Euclidean space, $\vartheta = \theta - \theta_r$ represents the temperature variation from the uniform reference temperature θ_r and α_0 is the initial value of α at the time $t = 0$. As a consequence the time derivative of the thermal displacement field is the temperature variation, i.e. $\dot{\alpha} = \vartheta$.

Let us now derive the complete set of the energy functions for the considered GN model and the related constitutive relations in the framework provided by convex/saddle functions. For simplicity, in the sequel, the dependence on the variable $\mathbf{x}$ is dropped and the thermal displacement gradient $\nabla\alpha$ is denoted by $\mathbf{g}$, i.e. $\mathbf{g} = \nabla\alpha$.

2.1. Conjugate of the free energy with respect to one state variable

A fundamental hypothesis in GN model is the introduction of the thermal displacement so that the thermal displacement