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Fluid Dynamics Research 39 (2007) 221–266
FLUID DYNAMICS RESEARCH

Local similarity of velocity distributions in homogeneous isotropic turbulence

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Received 19 December 2005; received in revised form 28 March 2006; accepted 19 August 2006

Communicated by T. Kambe

Abstract

Succeeding to the previous paper [Tatsumi and Yoshimura, 2004. Inertial similarity of velocity distributions in homogeneous isotropic turbulence. Fluid Dyn. Res. 35, 123–158] which dealt with the inertial similarity of the velocity distributions of homogeneous isotropic turbulence, the local similarity of the velocity distributions is investigated. The equations for the one- and two-point velocity distributions are expressed in the local dimensionless variables based on the mean energy-dissipation rate $\bar{\varepsilon}$ and the kinetic viscosity v and solved by making use of the cross-independence closure hypothesis [Tatsumi, 2001. Mathematical physics of turbulence. In: Kambe, T., et al. (Eds.), Geometry and Statistics of Turbulence. Kluwer Academic Publishers, Dordrecht, pp. 3–12]. The velocity distributions are obtained as continuous solutions in the local variables which coincide with the inertial normal distributions out of the local similarity range. The one-point velocity distribution, which was given by the normal distribution N1 with the parameter α_0 under the inertial similarity, is expressed in the local variables as the distribution N1 but with the local parameter $\alpha_0^*(=\alpha_0/v)$. The two-point velocity distribution is expressed in terms of the velocity-sum distribution and the velocity-difference distribution as before. The velocity-sum distribution, which was given by the normal distribution N2 with the parameter $\alpha_0/2$ under the inertial similarity, is obtained as the normal distribution N3 with the local parameter $\alpha_+^*(r^*)$ which changes with the local distance $r^*(=|\mathbf{r}^*|)$ between the two points. Since $\alpha_+^*(r^*)$ tends to $\alpha_0^*/2$, corresponding to $\alpha_0/2$ of N2, for $r^* \to \infty$ and to α_0^*, corresponding to α_0 of N1, for $r^* \to 0$, the distribution N3 satisfies the boundary conditions at both ends of the local similarity range. The velocity-difference distribution, which was given by the isotropic distribution N2 under the inertial similarity, becomes axi-symmetric with respect to $\mathbf{r}^*$ in the local range. The lateral distribution is obtained

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doi:10.1016/j.fluiddyn.2006.08.008

as the one-dimensional normal distribution N4 which satisfies the boundary conditions at both ends of the local range. The longitudinal distribution is obtained in three different similarity forms in the local range. It is expressed as the intermediate normal distribution N5 with the parameter $\alpha^*_{-0}(r^*)$ in the intermediate subrange, as the algebraic distribution A1 with the same $\alpha^*_{-0}(r^*)$ in the inertial subrange, and as the slightly asymmetric algebraic distribution A2 in the viscous subrange. The physical concepts of these results are discussed in comparison with existing experimental and numerical results.

Keywords: Local similarity; Cross-independence hypothesis; Homogeneous isotropic turbulence

1. Local similarity of turbulence

1.1. Inertial similarity and local similarity

It has been established in the previous paper (Tatsumi and Yoshimura, 2004), cited hereafter as I, that the one- and two-point velocity distributions of homogeneous isotropic turbulence are governed by the inertial similarity in the sense that the distributions depend upon only one parameter representing the mean energy-dissipation rate $\bar{\varepsilon}$ of turbulence and not the *viscosity* v of the fluid.

The one-point velocity distribution of the velocity $\mathbf{u}$ has been expressed by the normal distribution N1 (Eq. (I-67)) with the parameter $\alpha(=\bar{\varepsilon}/3)$. The two-point velocity distributions of the velocities $(\mathbf{u}_1, \mathbf{u}_2)$, written in terms of the distributions of the velocity-sum $\mathbf{u}_+ = (\mathbf{u}_1 + \mathbf{u}_2)/2$ and the velocity-difference $\mathbf{u}_- = (\mathbf{u}_2 - \mathbf{u}_1)/2$, have been expressed by the normal distribution N2 (Eqs. (I-87) and (I-97)) with the parameter $\alpha/2(=\bar{\varepsilon}/6)$ irrespective of the distance $r = |\mathbf{r}|$ between the two points.

On the other hand, the two-point velocity distributions must satisfy the coincidence conditions in the limit of $r \to 0$. The velocity-sum distribution N2 must tend in this limit to the distribution N1 as shown by Eq. (I-92), while the velocity-difference distribution N2 must tend to the delta distribution as shown by Eq. (I-107). Thus, the two-point velocity distributions which are expressed by N2 for all values of r must change discontinuously to the respective distributions in the limit of $r \to 0$. Such discontinuous change of the distributions is due to the inertial similarity associated with zero viscosity $v = 0$ and non-zero energy-dissipation rate $\bar{\varepsilon} > 0$, and actually the local similarity range associated with Kolmogorov's length $\eta = (v^3/\bar{\varepsilon})^{1/4}$ has been reduced to zero under this similarity.

At large but finite Reynolds numbers, the local similarity range has finite length-scale, so that we have to deal with the continuous change of the distributions in this range using the local coordinates based on the finite length-scale $\eta > 0$ under the finite viscosity $v > 0$. The situation concerning the energy-containing components of turbulence in the outer range $r > 0$ provides us with the boundary condition for the analysis in this local range.

1.2. Local similarity range and laminar boundary layer

In this context, it may be interesting to note a clear resemblance between the concepts of the local similarity range of turbulence, which has been proposed by Kolmogorov (1941) and also can be derived from the cross-independence closure hypothesis proposed by Tatsumi (2001), and the boundary layer of laminar flows due to Prandtl (1904), both representing small-scale domains surrounded by large-scale flows in the outer range either turbulent or laminar.