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Dynamic stress intensity factors for homogeneous and non-homogeneous materials using the interaction integral method

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ABSTRACT

An interaction integral is derived for the computation of dynamic stress intensity factors for both homogeneous and non-homogeneous materials. By selecting the appropriate auxiliary fields, the derived integrand does not involve any derivatives of material properties. Moreover, it can be proved that the integrand is still valid when the integral domain contains material interface. Therefore, the application of this method is more convenient. The numerical implementation of the interaction integral is combined with the extended finite element method without tip-enriched functions. Then, various homogeneous and non-homogeneous cracked bodies under dynamic loading are employed to verify the accuracy of the proposed technique.

1. Introduction

Dynamic stress intensity factors (DSIFs) are extremely important fracture parameters in understanding and predicting dynamic fracture behavior of a cracked body. Thus, an accurate evaluation of DSIFs is a crucial task for fracture mechanics researchers. Numerous methods have been developed for this purpose and the numerical method is one of the most convenient and reliable method for relatively complicated cases.

It has been found that the *J*-integral and its related techniques play an important role in static fracture mechanics, which can be referenced to the article written by Yu et al. [1]. In the case of dynamic fracture mechanics, Kishimoto et al. [2] developed a modified *J*-integral, which involves the inertial effects to determine DSIFs in conjunction with the finite element method (FEM). Similarly, Nishioka and Atluri [3] proposed another path independent *J*-integral to evaluate DSIFs for a crack subject to impact stress wave loading, and for a dynamically kinking as well as dynamically curving crack. The component separation method, proposed by Nishioka et al. [4], has been employed for mixed-mode problems in the calculation. However, the decomposition procedure is much complicated. To solve this problem, the interaction (energy) contour integral [5], which is derived from the *J*-integral by considering a composition of two admissible states (the actual fields and known auxiliary fields), may be a suitable choice. For the purpose of post-processing finite element solutions, the contour integrals are generally converted into equivalent domain forms. This operation removes the need to precisely capture the details of the singular fields near the crack tip. Song and Paulino [6] presented a domain formed interaction integral (conservation

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Nomenclature

Latin symbols

a	crack length
u_i, ε_{ij}, σ_{ij}	actual displacements, strains and stresses
u_i^{aux}, ε_{ij}^{aux}, σ_{ij}^{aux}	auxiliary displacements, strains and stresses
r, θ	polar coordinate
K_I, K_{II}	stress intensity factors corresponding to the actual fields
K_I^{aux}, K_{II}^{aux}	stress intensity factors corresponding to the auxiliary fields
$S_{ijkl}(x)$	compliance tensor
W	strain energy density
L	kinetic energy density
n_i	outward unit normal
$\dot{u}$, $\dot{u}^{aux}$	actual and auxiliary velocity
$\ddot{u}$	accelerated velocity
J^{aux}	energy release rate associated with the auxiliary fields
q	weight function
t_j	traction vector
E	Young's modulus
E_{tip}	Young's modulus at the crack tip
$f_\alpha(\mathbf{x})$	signed distance function
$N_I(\mathbf{x})$	finite element shape function
$H(\mathbf{x})$	Heaviside step function
$\mathbf{u}^h(\mathbf{x})$	displacement in an element
$\mathbf{x}$	field point
C_d	dilatational wave speed
t	time

Greek symbols

μ_0	local value of the shear modulus at the crack tip
ρ	mass density
v	Poisson's ratio
v_{tip}	Poisson's ratio at the crack tip
$\Gamma_0, \Gamma_B, \Gamma_C^+, \Gamma_C^-$	different integral path
$\Gamma_{interface}$	bi-material interface
(ξ_1, ξ_2)	curvilinear coordinates
$\varphi_J(\mathbf{x})$	shifted enrichment function for material interface
$\Psi_K(\mathbf{x})$	shifted enrichment function for crack
Δt	time step
β	material gradation parameter
Ω	domain
σ_0	amplitude of the impact loading

integral) to evaluate DSIFs for both homogeneous and non-homogeneous materials. In the process of derivation, the non-equilibrium formed auxiliary fields were employed, which have already been discussed by Kim and Paulino [7]. More recently, Réthoré et al. [8] presented an interaction integral based on Lagrangian conservation for the estimation of DSIFs for arbitrary 2D moving cracks. Most of the previous works are concerned with the materials with continuous and differentiable properties. If these conditions are not met, the application of the interaction integral method is impeded. Moreover, very few published papers have considered the cases that there are several material interfaces in the interaction integral domain. Actually, such phenomenon generally exists.

This paper is organized as follows. The derivation of the new formed interaction integral and its associated domain form without any derivatives of material properties is presented in Section 2. We also present the mathematically rigorous proof that the proposed interaction integral method is still valid even when there are material interfaces in the integral domain. Section 3 briefly introduces the extended finite element method without tip enriched functions. Section 4 provides the verification of the proposed method developed to evaluate DSIFs for both homogeneous and non-homogeneous materials by employing several benchmark problems. The dynamic fracture behaviors such as variation of DSIFs for various non-homogeneous crack bodies are also discussed in this section. Conclusions are presented in the last section.