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Analysis of cracked and un-cracked semicircular rings under symmetric loading

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ABSTRACT

Edge cracked specimens have been widely utilized for fracture testing. Edge cracked semicircular disk (ECSD) specimen has now been well characterized with regard to its form factor and weight function. This paper presents a modified semicircular ring version of this specimen to enhance the form factor in general while retaining other desirable features. The efficacy of the modified design is proved by combining theory of elasticity solutions with finite element results to arrive at the optimum design geometry. New insights emerging from this work are used to theoretically re-examine the arch-tension and the four-point bend specimens.

1. Introduction

Fracture testing of miniature glass and ceramic samples demands ingenious techniques and fixtures to obtain consistent results. In this regard, designing compression fracture specimens offers many advantages, as presented earlier by Surendra and Simha [1]. In particular, edge cracked semicircular disks (ECSD) exhibit some unique features with regard to the form factor for calculating stress intensity factors (SIFs). However, it is desirable to increase its stress intensity factor (SIF) values to promote testing tougher materials for the same overall specimen size. By adopting a semicircular ring geometry as shown in Fig. 8, we show that it is possible to not only enhance SIF but also accomplish accurate theoretical and numerical analysis to determining fracture parameters such as SIF, crack opening displacement (COD) and weight functions. The concept of weight function relates SIF to normal stress distribution in the corresponding uncracked body providing a way to understand the fracture behavior of the cracked body by analyzing corresponding uncracked body under given loading. Closed form elasticity solution of the uncracked semicircular ring (USR) for a given loading is therefore desirable to predict K_I of edge cracked semicircular ring (ECSR). USR geometry also offers an opportunity to compare results with classical elasticity solutions of stress distribution around a small semicircular hole in a large plate under uniform tension as one extreme case; and, as another elastic benchmark, we compare our results with the hyperbolic bending stress distribution in a curved beam subjected to end shear. These benchmarks have a special place in the long history of theory of elasticity, as given in the footnotes in the text by Timoshenko [2]. Most of the times, fatigue crack is initiated at a free surface where stress is concentrated. The inner boundary of USR acts as a semicircular edge notch where the normal stress is likely to concentrate under any symmetric loading. Closed form elasticity solution of USR can predict the stress concentration factor (SCF). Batista and Usenik [3] solved circular ring under diametral point loads at outer boundary by using power series in complex variable formulation

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Nomenclature

Symbols

a	length of edge crack
$\bar{a}_n$	relative crack length $(= a/(R_o - R_i))$
b	number of collocation points per boundary
c	distance of load holes in Arch-Tension specimen
$\bar{c}_i$	coefficients in form factor expressions
a_n, b_n, c_n, d_n and c'_n	Airy stress function coefficients
E	Young's modulus of the material $(= 3$ GPa$)$
$f(\theta), g(\theta), h(\theta)$ and $k(\theta)$	loading functions
f_σ	photoelastic material fringe constant
F	form factor or normalized mode-I SIF
h	thickness
K_I	mode-I stress intensity factor
m	number of quarter-point elements around crack-tip
M	bending moment
N	photoelastic fringe order
P	total compressive force applied on the disks
r, θ	polar coordinates
R_i, R_o	inner and outer radii
s	size of quarter-point elements
x, y	cartesian coordinates
β	load angle (and) flattened portion angle
ϕ	Airy stress function
v	Poisson's ratio of the material $(= 0.36)$
σ_0	far-field uniaxial tension
σ_1, σ_2	in-plane principal stresses
$\sigma_{rr}, \sigma_{\theta\theta}, \tau_{r\theta}$	2D stress components in polar coordinates
w	height of rectangular beam

Acronyms

AT	Arch-Tension
ECSD	edge cracked semicircular disk
ECSD(Ch)	edge cracked semicircular disk chopped at loaded portion
ECSR	edge cracked semicircular ring
ECSR(Ch)	edge cracked semicircular ring chopped at loaded portion
FE/FEM/FEA	finite element/finite element method/finite element analysis
FPB	four point bending
SCF	stress concentration factor
SEN	single edge notched specimen
SIF	stress intensity factor
USR	uncracked semicircular ring
USR(Ch)	uncracked semicircular ring chopped at loaded portion

and found SCF for different ratios of radii. Serati et al. [4] solved the ring under uniform pressure acting partially on inner and outer boundaries symmetric to an *in-plane* axis. Durelli and Lin [5] gave the stresses and displacements at the boundaries of a circular ring for different ratios of inner to outer radii obtained using Nelson's equations. Stampouloglou and Theotokoglou [6] considered semicircular cross tunnel, straight edges of which are constrained normally and curved edges under uniform pressure and determined the locus of *zero* hoop stress in the tunnel region.

Semi-infinite plate with a semicircular edge notch representing an extreme case of USR has a long history. Coker [7] predicted the SCF to be 2 based on an approximate solution by Leon. Mitchell [8] estimated it to be 3.08 by using a conformal mapping function of complex variable method. Ling [9,10] presented various estimates from different works, on SCF of half plane with semicircular edge notch, all of which are very close to 3.06 and calculated SCF to be 3.065336. Though many researchers solved for SCF of edge-notched half plane earlier, we show that the SCF can also be obtained using our present approach as accurately as Ling's result. Here, we assume a series form for Airy stress function in conjunction with boundary collocation to find the unknown coefficients of the assumed stress function. Similar to this approach, earlier, Fett [11] investigated edge cracked circular disk using boundary collocation method in a semi analytical manner. Yan Gu et al. [12] applied a boundary collocation method to solve 2D elasticity problems. In contrast, in this paper, boundary collocation is brought into the method for satisfying remaining boundary conditions only after satisfying