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Non-Fourier thermoelastic behavior of a hollow cylinder with an embedded or edge circumferential crack

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ABSTRACT

In this paper, the non-Fourier heat conduction theory is used to investigate the facture behavior of a hollow cylinder containing an embedded or edge circumferential crack under convective heat transfer boundary conditions. By neglecting the thermo-elastic coupling and inertial effects, the one-dimensional temperature field and the axial thermal stress for an un-cracked hollow cylinder are obtained in the Laplace domain. Then a mode I crack problem is formulated in the cylindrical system by using the superposition method. Integral transform technique is employed to reduce this mixed boundary value problem to a singular integral equation, which is solved numerically with the Gauss–Jacobi quadrature formulas. Finally, the effects of phase lag of heat flux, Biot's number, and crack geometry on the transient temperature field, axial stress, and stress intensity factors are analyzed. It is found that the *C–V* heat conduction model gives more conservative results than the Fourier model for the structure safe design against fracture under thermal loading.

1. Introduction

The well-known Fourier heat conduction theory is an early empirical law proposed by Fourier in 1807 based on experiments and investigations, which holds for many media in the usual temperature gradient range and presents an infinite wave propagation speed [1]. When it comes to applications undergoing large temperature gradient, the assumption that thermal disturbance will be felt instantaneously at distances infinitely far from its source becomes unacceptable. To circumvent this deficiency of Fourier law, Cattaneo [2] and Vernotte [3] modified the classical Fourier model by introducing a new material property called phase lag of heat flux or thermal relaxation time. The modified non-Fourier model, known as the *C–V* heat conduction model or the hyperbolic heat conduction model, results in a hyperbolic heat conduction equation and a finite thermal wave propagation speed.

Mitra et al. [4] gave the experimental evidence of the wave nature of heat propagation in the processed meat and demonstrated that the *C–V* heat conduction model was accurate to present the heat conduction process in biological materials. Kaminski [5] tested the values of phase lag of heat flux for some selected materials with nonhomogeneous inner structures, which were in the range of 10^1–10^2 s. The one-dimensional transient hyperbolic heat conduction in a functionally graded hollow cylinder was investigated by Babaei and Chen [6] using the Laplace transform technique. Torabi and Saedodin [7] studied the two-dimensional hyperbolic heat conduction problem in a finitely long solid cylinder with normal heat flux

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Nomenclature

a	inner radius of the crack
A_j	unknown coefficients
b	outer radius of the crack
b_{ij}	variables defined in Appendix B
C_i	unknown coefficients
C_{ij}	variables defined in Appendix A
C_v	specific heat capacity
d	thermal diffusivity
D_i	variables defined in Appendix A
E	Young's modulus
$E(\ )$	complete elliptic integral of the second kind
$f(\ )$	a bounded function in the fundamental solution of the normalized SIE
$F(\ )$	unknown function equals to $\phi(\ )$
$G(\ )$	a bounded function in the fundamental solution of SIE
h_i	convective heat transfer coefficient of the inner surface
h_o	convective heat transfer coefficient of the outer surface
$H(\ )$	Heaviside function
$I_n(\ )$	the nth-order modified Bessel functions of the first kind
k	thermal conductivity
k_a	stress intensity factor at the inner crack tip
k_b	stress intensity factor at the outer crack tip
$K(\ )$	complete elliptic integral of the first kind
$K_n(\ )$	the nth-order modified Bessel functions of the second kind
$L(\ )$	a function defined in Eq. (34)
$m(\ )$	a function defined in Eq. (35)
$M(\ )$	a function defined in Eq. (33)
p	axial stress determined for the un-cracked cylinder
$\vec{q}$	heat flux vector
r	radial coordinate
R	heat source
R_i	inner radius of the hollow cylinder
R_o	outer radius of the hollow cylinder
s	Laplace variable
t	time
T	temperature
T_i	temperature of the surrounding internal environment of the cylinder
T_o	temperature of the surrounding external environment of the cylinder
T_∞	initial temperature
u_r	radial displacement
u_z	axial displacement
x	integral variable
Y_i	variables defined in Appendix B
z	axial coordinate

Greek letters

α	coefficient of linear thermal expansion
β	a power law index in the fundamental solution of SIE
$\delta(\ )$	Dirac delta function
δT	temperature change
$\vec{\nabla}$	spatial gradient operator
Δ, Δ_j	determinants defined in Appendix B
η	integral variable
$\phi(\ )$	dislocation density function
$\Psi(\ )$	Love potential function
γ	a power law index in the fundamental solution of SIE
ξ	integral variable
λ	variable defined in Eq. (10)
μ	Poisson's ratio
ρ	normalized r'