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Application of Reddy's third-order theory to delaminated orthotropic composite plates

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ABSTRACT

In this work the third-order shear deformation theory by Reddy is applied and modified to analyze delaminated orthotropic composite plates. The delaminated plate portion is captured by Reddy's traditional theory, while a novel double-plate system is developed for the undelaminated part. It is shown that in the uncracked part four conditions are required to satisfy in symmetrically delaminated plates. The conditions involve the imposition of traction-free boundaries and the interface constraints. These four conditions enable the reduction of the parameters from nine to five in the displacement field. The governing equations show significant coupling among the stress resultants of the uncracked portion, that has to be considered in the continuity conditions between the delaminated and undelaminated parts. To demonstrate the application of the present model a simply-supported delaminated plate subjected to a concentrated force is analyzed. The distribution of the mode-II and mode-III energy release rates and their ratio are calculated using the 3-dimensional *J*-integral. The finite element model of the plate is also created using brick-type elements. The comparison of the analytical and finite element results shows very good agreement. It is shown that the deformations around the delamination front can be captured by the third-order plate theory with high accuracy.

1. Introduction

Laminated composite materials are used in many type of engineering structures (e.g. cars, airplanes, medical prosthetic devices, bridges, sports equipments, etc.). There is a large number of different damage modes of composite structures depending on the function, shape and loading conditions involved (Phillips, 1989; Tsai, 1992). In laminated composite structures the delamination or interlaminar fracture can take place as the result of quasi-static impact and the formation of cracks and notches during the application (Anderson, 2005). Therefore, the determination of the resistance against the delamination between the layers is a key step, as well. This property is characterized by the energy release rate (ERR) and its critical value (CERR) can be considered as the limit value in the course of proportionating composite structures. In accordance with linear elastic fracture mechanics (LEFM) there are three basic modes including mode-I, mode-II and mode-III fractures (Anderson, 2005). The interlaminar fracture toughness under mode-I (Sorensen, 2007; Sorensen et al., 2007; Jumel et al., 2011; Kim et al., 2011; Peng et al., 2011), mode-II (Arrese et al., 2010; Argüelles et al., 2011), mixed-mode I/II (Reeder and Crews, 1990; Bennati et al., 2009; Luo and Tong, 2009; Bennati et al., 2013a, b) is – in general – determined by cracked beam specimens. These samples have already been discussed in previous papers (Brunner and Flüeler, 2005; Brunner et al., 2008), therefore here we do not go into details in this respect. Considering the mode-III (Rizov et al., 2006; Szekrényes, 2009a; de Moura et al., 2009; de Morais and Pereira, 2009; de Morais et al., 2011; Pereira et al., 2011; Browning et al., 2010, 2011) and mixed-mode I/III (Pereira and de Morais, 2009; Szekrényes, 2009b), II/III (Szekrényes, 2007; de Morais and Pereira, 2008; Kondo et al., 2010, 2011; Nikbakht et al., 2010; Suemasu et al., 2010; Szekrényes, 2012a; Mladensky and Rizov, 2013) and I/II/III (Szekrényes, 2011; Davidson and Sediles, 2011) extensive reviews can be found in recent papers on the state-of-art situation of the available test methods for composite materials.

The present paper focuses essentially on the mixed-mode II/III interlaminar fracture in laminated composite plates. In the last few years several plate bending specimens were developed for the investigation of the mode-III (de Morais and Pereira, 2009; de Morais et al., 2011), mixed-mode I/III (Pereira and de Morais, 2009) and II/III (de Morais and Pereira, 2008) fractures in

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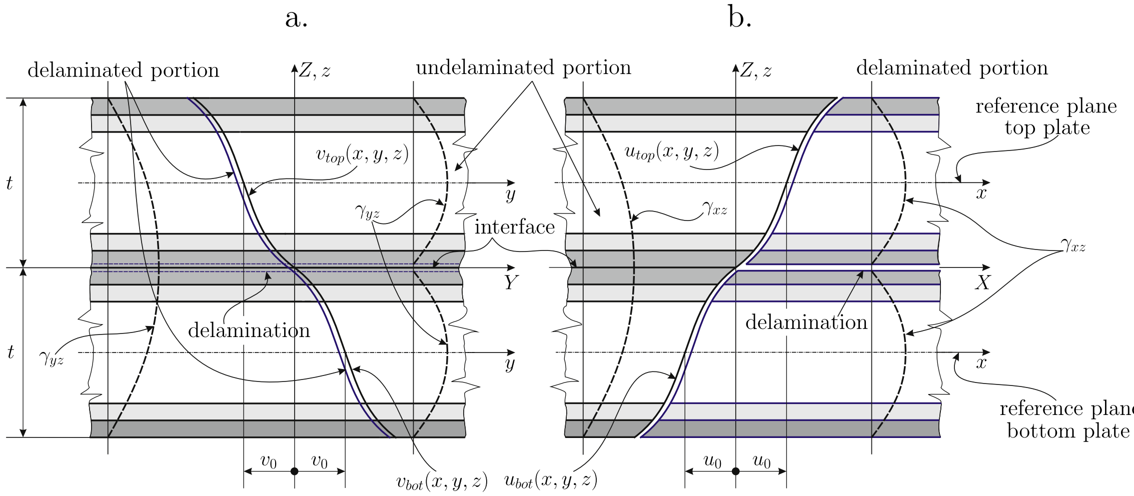

Fig. 1. Cross sections and deformations of the top and bottom plate elements of a delaminated plate in the *y*-*z* (a) and *x*-*z* (b) planes.

composites. In contrast with mode-I, mode-II and mixed-mode I/II tests, where simple beam samples are used, the main aspect of the former test methods is that plate shape specimens are used. This fact involves several difficulties compared to mode-I and mode-II tests. Analytical solutions are not yet available for plate bending tests, and so at present the finite element analysis is the only data reduction method for these tests. To calculate the ERR in numerical models there are several possibilities (Sankar and Sonik, 1995; Davidson et al., 2000; Park and Sankar, 2002; Bruno et al., 2005; Pereira and de Morais, 2009; Pereira et al., 2011), however each involves some difficulties. The most widely used method is the virtual crack closure technique (VCCT) (e.g.: Marat-Mendes and Freitas, 2010; Davidson and Sediles, 2011). This method requires a refined mesh in the vicinity of the delamination front and calculates the ERRs using the nodal forces and displacements. For a large model the refined mesh could result in very high element number. Without any doubt the method is very effective, however it is not yet implemented in most of the FE codes.

This paper addresses the development of an analytical solution to delaminated orthotropic composite plates based on Reddy's third-order theory (Reddy, 2004). The primary parameters of Reddy's theory are the inplane and transverse displacements and the rotations of the midplane normal. It was shown later that the performance of Reddy's theory can be improved by choosing the midplane shear strains instead of the rotations to be the primary parameters (Ren and Hinton, 1986). It was shown later by developing a plate finite element that the latter modification leads to accurate results with coarser mesh and less computation cost (Kulkarni and Kapuria, 2007). In some previous and current papers the application of the classical laminated plate theory (CLPT) (Szekrényes, 2012b, 2013b,a), first-order (FSDT) (Szekrényes, 2013a), second-order (SSDT) (Szekrényes, 2013c) and general third-order (TSDT) (Szekrényes, 2013d) shear deformation theories were applied to delaminated orthotropic plates under mixed-mode II/III condition. The *J*-integral (Rice, 1968; Cherepanov, 1997) for delaminated plates subjected to bending was derived and separated into mode-II and mode-III components. The distribution of the mode-II and mode-III ERRs as well as the mode ratio were determined along the delamination front of a simply-supported delaminated plate. The results were compared to those by a finite element model including the VCCT method and it was shown that plate analysis can provide reasonable accuracy. However, further improvement is possible by utilizing and modifying Reddy's third-order shear deformation theory, which is the main idea of this paper. It is shown that the complex deformations near the delamination front require the application of higher (e.g. third-) order theories in order to reach the sufficient accuracy from the fracture mechanical point of view.

Although in this paper we consider only symmetrically delaminated plates with a single crack, it is possible to extend the model to plates with several asymmetric interfacial cracks. The literature presents the solution of similar problems (including multiple delaminated plates) using the layerwise stress approach including plates subjected to uniaxial extension (Saeedi et al., 2012a, b), cylindrical bending (Saeedi et al., 2013a) and invariant load (Saeedi et al., 2013b). The latter works present very accurate results for the stress field compared to finite element calculations. In Reddy's theory an assumed displacement field is utilized, while in the layerwise approach there are no ad hoc assumptions. In the former it is not possible to predict the interlaminar normal stress, while in the latter it is. In spite of that it will be shown at the end of this paper that Reddy's theory gives a very accurate prediction for the distribution of ERRs along the crack front.

2. Reddy's third-order shear deformation theory

The third-order shear deformation theory by Reddy is applied to capture the deformation of the delaminated part of layered composite plates under mixed-mode II/III loading. The plate is shown by Fig. 1 consisting of the delaminated and undelaminated (or uncracked) portions. Reddy's third-order plate theory assumes third-order functions in terms of z (local through-thickness coordinate) for the in-plane displacement components of elastic plates. The solution satisfies the traction-free conditions at the top and bottom boundary surfaces of the plate (Reddy, 2004) by eliminating the parameter in the quadratic term, moreover the cubic term is related to the parameter of linear term and the plate deflection:

$$u(x,y,z)_\alpha = \pm u_0(x,y) + \theta_x(x,y)\cdot z - \frac{4z^3}{3t^2}\left(\theta_x + \frac{\partial w}{\partial x}\right) \tag{1}$$

$$v(x,y,z)_\alpha = \pm v_0(x,y) + \theta_y(x,y)\cdot z - \frac{4z^3}{3t^2}\left(\theta_y + \frac{\partial w}{\partial y}\right) \tag{2}$$