European Journal of Mechanics A/Solids 43 (2014) 25–43

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

Pre-kinking analysis of a constant moving crack in a magnetoelectroelastic strip under in-plane loading

Keqiang Hu [a,*], Zengtao Chen [a], Zheng Zhong [b]

[a] Department of Mechanical Engineering, University of New Brunswick, Fredericton, New Brunswick E3B 5A3, Canada
[b] School of Aerospace Engineering and Applied Mechanics, Tongji University, Shanghai 200092, PR China

ARTICLE INFO

Article history:
Received 6 February 2013
Accepted 22 August 2013
Available online 5 September 2013

Keywords:
Constant moving crack
Magnetoelctroelastic strip
Crack kinking

ABSTRACT

A constant moving crack in a magnetoelectroelastic strip under in-plane mechanical, electric and magnetic loading is considered for impermeable and permeable crack surface boundary conditions, respectively. Fourier transform is applied to reduce the mixed boundary value problem of the crack to dual integral equations, which are further transformed into Fredholm integral equations of the second kind. Steady state asymptotic fields near the crack tip are obtained and the corresponding field intensity factors are defined. The exact solution for a cracked infinite magnetoelectroelastic material can be recovered if the width of the strip tends to infinity. The crack speed and the geometric size of the strip affect the singular field distribution around the crack tip and the influences of electric and magnetic loading on the crack tip fields are discussed. The crack kinking phenomenon is investigated by applying the maximum hoop stress intensity factor criterion.

1. Introduction

Composite materials consisting of piezoelectric and piezomagnetic phases exhibit magnetoelectric effect that is not present in single-phase piezoelectric or piezomagnetic materials. Owing to the unique magnetoelectroelastic coupling effect, these materials can be used in intelligent structures as sensors and actuators. Studies on the properties of piezoelectric/piezomagnetic composites have drawn considerable attention in recent years. Some defects (such as dislocations and cracks) could be induced during the manufacturing processes or during service by the mechanical, electric or magnetic loading, which can adversely influence the performance of the structures. Consequently, it is necessary to develop our understanding of the characteristics of magnetoelectroelastic materials with defects.

In recent decades, there has been a growing interest among researchers in solving fracture mechanics problems in magnetoelectroelastic media. Crack initiation behavior in a magnetoelectroelastic composite under in-plane deformation was investigated by Song and Sih (2003). Qin (2005) obtained 2D Green's functions of defective magnetoelectroelastic solids under thermal loading, which can be used to establish boundary formulation and to analyze relevant fracture problems. The singular magnetoelectroelastic fields in a cracked rectangular piezoelectromagnetic body were obtained by Hu et al. (2006a). The dynamic response of a penny-shaped crack in a magnetoelectroelastic layer was studied by Feng et al. (2007). Wang and Mai (2007) discussed the different electromagnetic boundary conditions on the crack-faces in magnetoelectroelastic materials, which possess coupled piezoelectric, piezomagnetic and magnetoelectric effects. Zhong and Li (2007) gave a magnetoelectroelastic analysis for an opening crack in a piezoelectromagnetic solid. Zhou and Chen (2008) analyzed a partially conducting, mode I crack in a piezoelectromagnetic material. Zhao and Fan (2008) proposed a strip, electric-magnetic breakdown model in a magnetoelectroelastic medium to study the nonlinear character of electric field and magnetic field on fracture of magnetoelectroelastic materials. The problem of a planar magnetoelectroelastic, layered half-plane subjected to generalized line forces and edge dislocations is analyzed by Ma and Lee (2009). Li and Lee (2010) established fundamental solutions for in-plane magnetoelectroelastic governing equations and studied collinear, unequal cracks in magnetoelectroelastic materials. Wan et al. (2012) investigated a mode III crack crossing the magnetoelectroelastic, bimaterial interface under concentrated magnetoelectromechanical loads. Hu and Chen (2012a) investigated the dynamic response of a cracked, magnetoelectroelastic layer sandwiched between dissimilar elastic layers under anti-plane deformation by the integral transform method.

* Corresponding author. Tel./fax: +1 506 458 7104.
E-mail addresses: ckhu@unb.ca, keqianghu@163.com (K. Hu).

0997-7538/$ – see front matter
http://dx.doi.org/10.1016/j.euromechsol.2013.08.005

Pre-curving analysis of an opening crack in a magnetoelectroelastic strip under in-plane impact loadings is given by Hu and Chen (2012b). An efficient numerical model based on dual boundary element method (BEM) was presented by Rojas-Díaz et al. (2012) to analyze different crack face boundary conditions in 2D magneto-electroelastic media.

Theoretical investigation of crack propagation in elastic materials began with Yoffe's (1951) analysis of the near-tip field of a constant moving crack, and some of the subsequent investigations were carried out by Craggs (1960), Freund (1972), Willis (1973), Freund (1990), Yang et al. (1991), among others. Gao (1993) proposed a wavy-crack model to explain some important discrepancies existing between theories and experiments, and the analysis indicates that the basic mechanism of dynamic branching is somewhat like a thermally activated kinetic process.

Considering the coupling effect of mechanical and electrical fields, the moving crack problem in a piezoelectric material under longitudinal shear has been studied by Chen and Yu (1997), Chen et al. (1998), Li et al. (2000), Kwon and Lee (2001), etc. Hu and Zhong (2005) considered a moving mode-III crack in a functionally graded piezoelectric strip and showed that the gradient of the material properties can affect the magnitudes of the stress intensity factors. Under the assumption of in-plane, electro-mechanical loadings, the moving crack problems in piezoelectric materials have been investigated by Soh et al. (2002), Herrmann and Loboda (2006), Piva et al. (2007), etc.

The moving crack problem in an infinite size, magneto-electroelastic body under anti-plane shear and in-plane electro-magnetic loading has recently been solved by Hu and Li (2005a), and the results predicted that the moving crack may curve when the velocity of the crack is greater than a certain value. Tian and Rajapakse (2008) presented a theoretical study for crack branching in magnetoelectroelastic solids by extending the generalized dislocation model. The moving crack at the interface between dissimilar magnetoelectroelastic materials has been investigated by Hu et al. (2006b) and Zhong and Li (2006). Tupholme (2009) studied a moving anti-plane shear crack in a transversely isotropic magnetoelectroelastic medium when subjected to representative, non-constant crack-face loading conditions. A constant moving crack in an infinite magnetoelectroelastic medium under in-plane mechanical, electric and magnetic loadings was investigated for impermeable crack surface boundary conditions by Hu and Chen (2013).

To the best knowledge of the authors, no results on the moving crack in a magnetoelectroelastic strip with finite width under in-plane magnetoelectroelastic loading have been reported in the literature. This problem is solved in this paper. Fourier transforms are applied and the mixed boundary value problem of the crack is reduced to solving dual integral equations, which are further transformed into Fredholm integral equations of the second kind. The asymptotic fields near the crack tips are obtained and the corresponding field intensity factors are defined. The crack kinking phenomenon is investigated by applying the maximum hoop stress intensity factor criterion. The coupled magnetoelectroelastic effects on the crack-tip fields are investigated and the influences of the crack speed and the finite size of the strip on the dynamic fracture behavior are discussed.

2. Basic equations for magnetoelectroelastic material

Consider a linear magnetoelectroelastic material which is assumed to be transversely isotropic and denote the Cartesian coordinates of a point by x_j (j = 1,2,3). The dynamic equilibrium equations are given as

$$\sigma_{ij,i} + f_j = \rho \frac{\partial^2 u_j}{\partial t^2}, \quad D_{i,i} - f_e = 0, \quad B_{i,i} = 0 \tag{1}$$

where σ_{ij}, u_j, D_i and B_i are the components of stress, displacement, electrical displacement and magnetic induction, respectively; f_j and f_e are the body force and electric charge density, respectively; ρ is the mass density of the magnetoelctroelastic material; a comma followed by i (i = 1,2,3) denotes partial differentiation with respect to the coordinate x_i, and the summation convention over repeated indices is applied. The constitutive equations can be written as

$$\begin{aligned} \sigma_{ij} &= C_{ijks}\varepsilon_{ks} - e_{sij}E_s - h_{sij}H_s \\ D_i &= e_{iks}\varepsilon_{ks} + \lambda_{is}E_s + d_{is}H_s \\ B_i &= h_{iks}\varepsilon_{ks} + d_{is}E_s + \gamma_{is}H_s \end{aligned} \tag{2}$$

where ε_{ks}, E_s and H_s are components of strain, electric field and magnetic field; C_{ijks}, e_{iks}, h_{iks} and d_{is} are elastic, piezoelectric, piezomagnetic and electromagnetic constants; λ_{is} and γ_{is} are dielectric permitivities and magnetic permeabilities, respectively. The following reciprocal symmetries hold:

$$\begin{aligned} &C_{ijks} = C_{jiks} = C_{ijsk} = C_{ksij}, \quad e_{sij} = e_{sji} \\ &h_{sij} = h_{sji}, \quad d_{ij} = d_{ji}, \quad \lambda_{ij} = \lambda_{ji}, \quad \gamma_{ij} = \gamma_{ji} \end{aligned} \tag{3}$$

The gradient equations are

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad E_i = -\phi_{,i}, \quad H_i = -\varphi_{,i} \tag{4}$$

where u_i is the displacement vector, ϕ and φ are the electric and magnetic potentials, respectively.

Under the assumption of plane strain, the constitutive equations take the form as (Huang and Kuo, 1997):

$$\begin{aligned}
\begin{Bmatrix} \sigma_{11} \\ \sigma_{33} \\ \sigma_{13} \end{Bmatrix} &= \begin{bmatrix} C_{11} & C_{13} & 0 \\ C_{13} & C_{33} & 0 \\ 0 & 0 & C_{44} \end{bmatrix} \begin{Bmatrix} u_{1,1} \\ u_{3,3} \\ u_{1,3}+u_{3,1} \end{Bmatrix} + \begin{bmatrix} 0 & e_{31} \\ 0 & e_{33} \\ e_{15} & 0 \end{bmatrix} \begin{Bmatrix} \phi_{,1} \\ \phi_{,3} \end{Bmatrix} + \begin{bmatrix} 0 & h_{31} \\ 0 & h_{33} \\ h_{15} & 0 \end{bmatrix} \begin{Bmatrix} \varphi_{,1} \\ \varphi_{,3} \end{Bmatrix} \\
\begin{Bmatrix} D_1 \\ D_3 \end{Bmatrix} &= \begin{bmatrix} 0 & 0 & e_{15} \\ e_{31} & e_{33} & 0 \end{bmatrix} \begin{Bmatrix} u_{1,1} \\ u_{3,3} \\ u_{1,3}+u_{3,1} \end{Bmatrix} - \begin{bmatrix} \lambda_{11} & 0 \\ 0 & \lambda_{33} \end{bmatrix} \begin{Bmatrix} \phi_{,1} \\ \phi_{,3} \end{Bmatrix} - \begin{bmatrix} d_{11} & 0 \\ 0 & d_{33} \end{bmatrix} \begin{Bmatrix} \varphi_{,1} \\ \varphi_{,3} \end{Bmatrix} \\
\begin{Bmatrix} B_1 \\ B_3 \end{Bmatrix} &= \begin{bmatrix} 0 & 0 & h_{15} \\ h_{31} & h_{33} & 0 \end{bmatrix} \begin{Bmatrix} u_{1,1} \\ u_{3,3} \\ u_{1,3}+u_{3,1} \end{Bmatrix} - \begin{bmatrix} d_{11} & 0 \\ 0 & d_{33} \end{bmatrix} \begin{Bmatrix} \phi_{,1} \\ \phi_{,3} \end{Bmatrix} - \begin{bmatrix} \mu_{11} & 0 \\ 0 & \mu_{33} \end{bmatrix} \begin{Bmatrix} \varphi_{,1} \\ \varphi_{,3} \end{Bmatrix}
\end{aligned} \tag{5}$$