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# Thermal vibration and transient response of magnetostrictive functionally graded material plates

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ABSTRACT

The results of functionally graded material (FGM) plate with mounted magnetostrictive layer are investigated in thermal vibration and transient response by using the generalized differential quadrature (GDQ) method. The modified shear correction coefficient can be obtained based on the total strain energy equivalence principle. The computational real value solutions of Terfenol-D FGM plate with four edges in simply supported boundary conditions are obtained for the center displacement. Some parametric effects on the Terfenol-D FGM plates are analyzed, there are: shear correction coefficient values, thickness of mounted magnetostrictive layer, control gain values, temperature of environment and power law index of FGM. The effect of different mechanical boundary conditions on the results of numerical GDQ method is also investigated.



## 1. Introduction

The functionally graded material (FGM) can be used as a thermal barrier in the application field of aircraft, space vehicles, reactor vessels and engineering. There are some computational methods in analyzing laminated composite plates and structures made up of FGM. In 2013, Phan-Dao et al. (2013) presented an edge-based smoothed finite element method (ES-FEM) to obtain the well solutions of free vibration and buckling for laminated composite plates. In 2013, Valizadeh et al. (2013) introduced nonuniform rational B-spline (NURBS) based iso-geometric finite element method (FEM) to obtain the static and dynamic numerical results of FGM plates. In 2012, Thai et al. (2012) used NURBS based iso-geometric approach to present the static, free vibration, and buckling numerical results of laminated composite plates. In 2012, Natarajan et al. (2012) presented the numerical solutions of size dependent linear free flexural vibration of NURBS based iso-geometric FEM for FGM nanoplates. In 2011, Baiz et al. (2011) used the quadrilateral element with smoothed curvatures (SFEM) and the extended finite element method (XFEM) to simulate the linear buckling isotropic cracked plates. In 2011, Ootao et al. (2011) used piezoelectric and magnetostrictive materials as the FGM strip to calculate the transient thermal stress. Terfenol-D ($Tb_{0.3}D_{0.7}Fe_{1.9}$) made in USA is one of commercial magnetostrictive materials in the world. In 2010, Arunanidhi and Singaperumal (2010) presented the Terfenol-D rod material used in design of magnetostrictive actuator. In 2010, Valadkhan et al. (2010) presented a load-dependent hysteresis modeling technique at different loads for high bandwidth magnetostrictive actuators. In 2009, Jia et al. (2009) presented the Terfenol-D rod material used in the application of digital PID control system of magnetostrictive actuator. In 2008, Olabi and Grunwald (2008) showed the Young's modulus of Terfenol-D material varies almost linearly with the external magnetic field. In 2006, Chi and Chung (2006) presented the mechanical behavior of FGM plate under transverse load. In 2004, Huang and Shen (2004) investigated the nonlinear dynamic vibration response of four edges simply supported FGM plates in thermal environments.

The author has some generalized differential quadrature (GDQ) experiences in the study of magnetostrictive Terfenol-D material plate. In 2012, Hong (Hong, 2012) presented the thermal vibration of magnetostrictive FGM plate under rapid heating on its lower surface. In 2010, Hong (Hong, 2010) presented the transient responses of magnetostrictive plates. In this GDQ study of Terfenol-D FGM plate subjected to thermal loading in sinusoidal functions of time, displacement and temperature with four sides in simply supported boundary conditions, center displacements were obtained under uncontrolled/controlled gain. Some parametric effects on the Terfenol-D FGM plate were analyzed, there are: shear correction coefficient values, thickness of mounted magnetostrictive layer, control gain values, temperature of environment and power law index of FGM. The effects of different mechanical boundary condition: four sides clamp on the results of numerical GDQ method are also investigated.

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## 2. Formulation

### 2.1. Functionally graded material properties

The material properties of power-law function two-material FGM plate are continuous change across its thickness. It is reasonable to consider the Young's modulus of FGM plate $E_{fgm}$ as the typical strong property (great value in $GP_a$ unit) for all the material properties, and the others material properties: $\nu_{fgm}$, $\rho_{fgm}$, $\alpha_{fgm}$, $\kappa_{fgm}$ and $C_{vfgm}$ of FGM plate are assumed in the average form for the simply direct stiffness integration as introduced by Chi and Chung (2006) as follows, although for the numerical analysis of an investigation on more general form material properties case is feasible.

$$E_{fgm} = (E_2 - E_1)\left(\frac{z + h/2}{h}\right)^{R_n} + E_1. \tag{1a}$$

$$\nu_{fgm} = (\nu_2 + \nu_1)/2, \tag{1b}$$

$$\rho_{fgm} = (\rho_2 + \rho_1)/2, \tag{1c}$$

$$\alpha_{fgm} = (\alpha_2 + \alpha_1)/2, \tag{1d}$$

$$\kappa_{fgm} = (\kappa_2 + \kappa_1)/2, \tag{1e}$$

$$C_{vfgm} = (C_{v2} + C_{v1})/2. \tag{1f}$$

where $z$ is the axis coordinate in the thickness direction, $h$ is the thickness of FGM plate, $R_n$ is the power law index. $E_1$ and $E_2$ are the Young's modulus, $\nu_1$ and $\nu_2$ are the Poisson's ratios, $\rho_1$ and $\rho_2$ are the densities. $\alpha_1$ and $\alpha_2$ are the thermal expansion coefficients, $\kappa_1$ and $\kappa_2$ are the thermal conductivities, $C_{v1}$ and $C_{v2}$ are the specific heats of the FGM constituent material 1 and 2, respectively. $E_1$, $E_2$ terms and other material properties can be expressed corresponding to term individual property $P_i$ equation consists with the temperature coefficients and the temperature of environment $T$ by Reddy and Chin in 1998 (Reddy and Chin, 1998).

### 2.2. Displacements

The displacement components: $u$ $v$ and $w$ are assumed in the well-know first-order shear deformation theory (FSDT) model as follows:

$$u = u^0(x,y,t) + z\psi_x(x,y,t), \tag{2a}$$

$$v = v^0(x,y,t) + z\psi_y(x,y,t), \tag{2b}$$

$$w = w(x,y,t). \tag{2c}$$

where $u^0$, $v^0$ and $w$ are displacements of the middle-plane in the $x$, $y$ and $z$ axes direction of plate, respectively, $\psi_x$ and $\psi_y$ are the shear rotations, $t$ is time.

### 2.3. GDQ method review

The GDQ implementation was presented by Shu and Richards in 1990. The GDQ method approximates the derivative of functions, it is restated: the derivative of a smooth function at a discrete point can be discretized by an approximated weighting linear sum of the function values at all the discrete points in the derivative direction by Shu and Du in 1997 (Shu and Du, 1997).

### 2.4. Thermo-elastic and magnetostrictive stress-strain relations

Usually the magnetostrictive FGM plate as shown in Fig. 1 is used in the ultimate high thermal environment, the temperature difference $\Delta T = T_0(x,y,t) + z/h^* T_1(x,y,t)$ between the magnetostrictive FGM plate and curing area is considered, in which $h^*$ is the total thickness of magnetostrictive layer and FGM plate, $T_0(x,y,t)$ is the temperature before thermal loading, $T_1(x,y,t)$ is the temperature after thermal loading on the FGM plate. The stresses in the $k^{th}$ layer of the magnetostrictive FGM plate under the purely thermal temperature difference $\Delta T$ effect are given in the stress-strain equations by Lee and Reddy in 2005 (Lee and Reddy, 2005) as follows.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_{(k)} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_{(k)} \begin{Bmatrix} \varepsilon_x - \alpha_x \Delta T \\ \varepsilon_y - \alpha_y \Delta T \\ \varepsilon_{xy} - \alpha_{xy} \Delta T \end{Bmatrix}_{(k)} - \begin{bmatrix} 0 & 0 & \tilde{e}_{31} \\ 0 & 0 & \tilde{e}_{32} \\ 0 & 0 & \tilde{e}_{36} \end{bmatrix}_{(k)} \begin{Bmatrix} 0 \\ 0 \\ \tilde{H}_z \end{Bmatrix}_{(k)} \tag{3a}$$

$$\begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix}_{(k)} = \begin{bmatrix} \overline{Q}_{44} & \overline{Q}_{45} \\ \overline{Q}_{45} & \overline{Q}_{55} \end{bmatrix}_{(k)} \begin{Bmatrix} \varepsilon_{yz} \\ \varepsilon_{xz} \end{Bmatrix}_{(k)} - \begin{bmatrix} \tilde{e}_{14} & \tilde{e}_{24} & 0 \\ \tilde{e}_{15} & \tilde{e}_{25} & 0 \end{bmatrix}_{(k)} \begin{Bmatrix} 0 \\ 0 \\ \tilde{H}_z \end{Bmatrix}_{(k)} \tag{3b}$$

where $\sigma_x$ and $\sigma_y$ are the normal stresses. $\sigma_{xy}$, $\sigma_{yz}$ and $\sigma_{xz}$ are the shear stresses. $\alpha_x$ and $\alpha_y$ are the coefficients of thermal expansion, $\alpha_{xy}$ is the coefficient of thermal shear. $\overline{Q}_{ij}$ is the reduced stiffness of magnetostrictive FGM plate. $\varepsilon_x$, $\varepsilon_y$ and $\varepsilon_{xy}$ are in-plane strains. $\varepsilon_{yz}$ and $\varepsilon_{xz}$ are transverse shear strains. $\tilde{e}_{ij}$ is the transformed magnetostrictive coupling modulus. $\tilde{H}_z$ is the magnetic field intensity.

The simple forms of $\overline{Q}_{ij}$ are used for the FGM in 2007 by Shen (2007) as follows.

$$\overline{Q}_{11} = \overline{Q}_{22} = \frac{E_{fgm}}{1 - \nu_{fgm}^2}, \tag{4a}$$

$$\overline{Q}_{12} = \frac{\nu_{fgm} E_{fgm}}{1 - \nu_{fgm}^2}, \tag{4b}$$

$$\overline{Q}_{44} = \overline{Q}_{55} = \overline{Q}_{66} = \frac{E_{fgm}}{2\left(1 + \nu_{fgm}\right)}, \tag{4c}$$

$$\overline{Q}_{16} = \overline{Q}_{26} = \overline{Q}_{45} = 0. \tag{4d}$$

The simple forms of $\overline{Q}_{ij}$ for the magnetostrictive layer are used as follows.

$$\overline{Q}_{11} = \overline{Q}_{22} = E_{11}, \tag{4e}$$

$$\overline{Q}_{44} = \overline{Q}_{55} = \overline{Q}_{66} = E_{11}/2, \tag{4f}$$

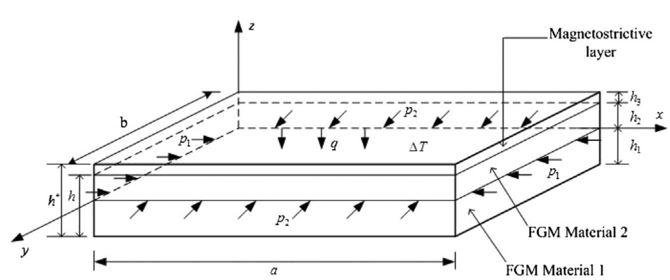


**Fig. 1.** Two-material FGM plate with magnetostrictive layer.