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Isogeometric analysis of laminated composite and sandwich plates using a new inverse trigonometric shear deformation theory

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ABSTRACT

This paper presents a new inverse tangent shear deformation theory (ITSDT) for the static, free vibration and buckling analysis of laminated composite and sandwich plates. In the present theory, shear stresses are vanished at the top and bottom surfaces of the plates and shear correction factors are no longer required. A weak form of the static, free vibration and buckling models for laminated composite and sandwich plates based on ITSDT is then derived and is numerically solved using an isogeometric analysis (IGA). The proposed formulation requires C^1-continuity generalized displacements and hence basis functions used in IGA fulfill this requirement. Numerical examples are provided to show high efficiency of the present method compared with other published solutions.

1. Introduction

In the past few decades, developments in science and technology have created motivations for researchers to find on new structural materials such as composite and sandwich. These materials have been used in various engineering disciplines such as aerospace engineering, automotive engineering, civil engineering, etc. Plates are an important part of many structures. Laminated composite plates are often made of several orthotropic layers and bonded together to achieve superior properties such as high stiffness and strength-to-weight ratios, long fatigue life, wear resistance, lightweight, etc. Especially, for sandwich plates, inner layers are replaced by a core which has low stiffness. Therefore, a good understanding of bending behavior, stress distribution, dynamic and buckling responses of the plates is necessary for researchers and users.

Several laminated plate theories have been investigated for composite and sandwich plates. The classical laminate plate theory (CLPT) (Bose and Reddy, 1998) is only suitable for thin plates. The first-order shear deformation theory (FSDT) (Whitney and Pagano, 1970), which shear deformation effect is regarded, can be applied for both moderately thick and thin plates. The FSDT does not satisfy free boundary conditions on the lower and upper surface of the plates, and hence shear correction factors need to be involved. To avoid using shear correction factors, many higher order shear deformation theories have been devised by the researchers, e.g., Ambartsumian (1958), Reissner (1975), Levinson (1980), Reddy (1984), Soldatos (1992), Karama et al. (2003) and Aydogdu (2009), etc. Classically, first-order and higher-order theories are used the equivalent single-layer models (ESL), which consider the same degrees of freedom for all laminate layers. In addition, several other equivalent-single-layer models for laminated plates have been proposed accounting for zig–zag effects and fulfillment of interlaminar continuity. Among these the one by Mau (1973), Chou and Corleone (1973), Di Sciuva (1987), Toledano and Murakami (1987), Ren (1986) and Castro et al. (2010) are herein mentioned. Mixed layer-wise and equivalent-single-layer theories based on Reissener Mixed Variational Theorem have been discussed by Carrera (1996, 1998, 2001). A historical review encompassing early and recent developments of advanced theories for laminated beams, plates and shells was revisited in (Carrera, 2003). Interested readers are addressed to that last paper for a more complete review on relevant topics.

In the effort to development of advanced computational methodologies, Hughes et al. (2005) have recently proposed an isogeometric analysis (IGA) that bridges the gap between

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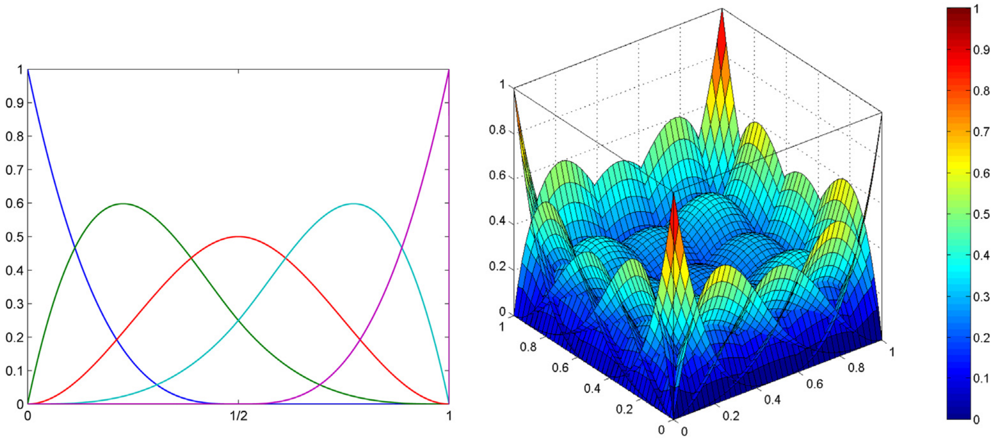

(a) 1D (b) 2D

Fig. 1. 1D and 2D cubic B-spline basis functions.

Computer Aided Design (CAD) and Finite Element Analysis (FEA). It means that the IGA uses basis functions generated from Non-Uniform Rational B-Splines (NURBS) in order to describe both the geometry and the unknown variables of the problem. Therefore, the process of meshing in IGA can be omitted and the two models for CAD and FEA integration into one. The main advantages of IGA are ability to represent exactly domains being conic sections and higher order approximation with arbitrarily high smoothness. In IGA, the exact geometry is maintained at the coarsest level of discretization and re-meshing is performed on this level without any further communication with CAD geometry. Furthermore, B-splines (or NURBS) provide a flexible way to perform refinement (or h-refinement), and degree elevation (Cottrell et al., 2007). Isogeometric analysis has been applied to a wide range of practical mechanics problems such as structural vibrations (Cottrell et al., 2006), nearly incompressible linear and nonlinear problems (Elguedj et al., 2008), structural shape optimization (Wall et al., 2008), Kirchhoff–Love shell (Kiendl et al., 2009, 2010; Nguyen-Thanh et al., 2011), isotropic Reissner–Mindlin shell (Benson et al., 2010), laminated composite/functionally graded plates based on FSDT (Thai et al., 2012a; Valizadeh et al., 2013; Kapoor and Kapania, 2012)/HSDT (Thai et al., 2012b; Tran et al., 2013, Nguyen-Xuan et al., 2013), laminated composite plates based on the layer-wise theory (Thai et al., 2013) and rotation-free shells (Benson et al., 2011), etc.

In this paper, an effectively approximate formulation based on a NURBS-based isogeometric analysis associated with a new inverse tangent shear deformation theory (ITSDT) is presented for static, free vibration and buckling analysis of laminated composite and sandwich plates. An inverse tangent function can be expressed by means of Taylor expansion, that has more general form than the classical polynomial. Generalized displacements are constructed using the NURBS basis functions that can yield higher-order continuity and fulfill easily the requirement of C^1-continuity of the HSDT models. Several numerical examples are illustrated to show high effectiveness of the present method. Obtained results are well compared with exact three-dimensional elasticity, analytical or semi-analytical and other numerical solutions.

The paper is arranged as follows: a brief review on the B-spline and NURBS surface is described in Section 2. Section 3 presents a formulation of a NURBS-based isogeometric analysis for composite sandwich plates. Several numerical examples are provided in Section 4. Finally we close our paper with some concluding remarks.

2. A brief review of NURBS functions and surfaces

2.1. Knot vectors and basis functions

Let $\Xi = [\xi_1, \xi_2, \ldots, \xi_{n+p+1}]$ be a nondecreasing sequence of parameter values, $\xi_i \le \xi_{i+1}, i = 1, \ldots, n+p$. The ξ_i are called knots, and Ξ is the set of coordinates in the parametric space. If all knots are equally spaced the knot vector is called uniform. If the first and the last knots are repeated $p+1$ times, the knots vector is described as open. A B-spline basis function is C^∞ continuous inside a knot span and C^{p-1} continuous at a single knot. A knot value can appear more than once and is then called a multiple knot. At a knot of multiplicity k the continuity is C^{p-k}. Given a knot vector, the B-spline basis functions $N_{i,p}(\xi)$ of order $p = 0$ are defined as follows

$$N_{i,0}(\xi) = \begin{cases} 1 & \xi_i \le \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

The basis functions of order $p > 0$ is defined by the following recursion formula (Piegl and Tiller, 1997)

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi) \quad \text{with} \quad p = (1,2,3,\ldots) \tag{2}$$

For $p = 0$ and 1, the basis functions of isogeometric analysis are identical to those of standard piecewise constant and linear finite elements, respectively. In IGA, the basis functions with $p \ge 2$ are considered (Hughes et al., 2005). Fig. 1 illustrates a set of one-dimensional and two-dimensional cubic B-spline basis functions for open uniform knot vectors $\Xi = \{0,0,0,0,½,1,1,1,1\}$.

2.2. NURBS surface

The B-spline curve is defined as

$$\mathbf{C}(\xi) = \sum_{i=1}^{n} N_{i,p}(\xi)\mathbf{P}_i \tag{3}$$

where $\mathbf{P}_i$ are the control points, n denotes the number of control points and $N_{i,p}(\xi)$ is the p^{th}-degree B-spline basis function defined on the open knot vector.

Given two knot vectors $\Xi = \{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\}$ and $\mathscr{H} = \{\eta_1, \eta_2, \ldots, \eta_{m+q+1}\}$ and a control net $\mathbf{P}_{i,j}$, a tensor-product B-spline surface is defined as