European Journal of Mechanics A/Solids 35 (2012) 1–9

Contents lists available at SciVerse ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

Nonlinear free vibration of functionally graded shear deformable sector plates by a curved triangular *p*-element

S.A. Belalia*, A. Houmat

Department of Mechanical Engineering, Faculty of Engineering, University of Tlemcen, B.P. 230, Tlemcen 13000, Algeria

ARTICLE INFO

Article history:
Received 10 December 2009
Accepted 16 January 2012
Available online 28 January 2012

Keywords:
Geometrically nonlinear free vibration
Functionally graded material
Mindlin plate theory
Sector plate
Curved triangular *p*-element

ABSTRACT

A *p*-version of the finite element method based on a curved triangular *p*-element is developed and applied to nonlinear free vibration analysis of functionally graded sector plates. The material is assumed to be temperature dependent and graded in the thickness direction according to the power-law distribution in terms of volume fractions of the constituents. In the geometrically nonlinear formulation, the Von Karman assumptions with Mindlin first-order shear deformation theory are used. The shape functions are constructed from the shifted Legendre orthogonal polynomials. The curved edge of the sector plate is represented accurately using the blending function method. The nonlinear equation of motion is obtained using the harmonic balance method and solved iteratively using the linearized updated mode technique. The linear and nonlinear frequencies are calculated for a functionally graded SUS304/Si_3N_4 clamped circular plate. The accuracy of the proposed method is demonstrated through convergence and comparison studies. Sector plates made out of three types of functionally graded materials (SUS304/Si_3N_4, AL/AL_2O_3, AL/ZrO_2) are considered. The effects of sector angle, thickness, and volume fraction exponent on the hardening behavior of a clamped sector plate are also investigated. It is shown that the increase or decrease of the hardening behavior depends upon these parameters.

1. Introduction

The use of Functionally Graded Materials (FGMs) in modern industries has recently gained considerable attention from researchers, especially in high temperature environments such as in nuclear plants and high speed spacecrafts. FGMs are special composites which are microscopically inhomogeneous. The material properties of the functionally graded plates are assumed to vary continuously through the thickness. The first concept of FGMs began in 1984 in Japan (Koizumi, 1993, 1997).

Many researchers have focused their studies on static and dynamic behaviors of FG plates under several effects. Reddy and Chin (1998) analyzed the dynamic thermoelastic response of functionally graded cylinders and plates. In the same year, Praveen and Reddy (1998) investigated the nonlinear transient thermoelastic analysis of FG ceramic-metal plates using a finite element formulation. The Axisymmetric bending and stretching of functionally graded solid and annular circular plates were studied by Reddy et al. (1999) using the first-order shear deformation theory. Reddy (2000) presented a theoretical formulation and finite element models for FG plates based on the third-order shear deformation theory. The nonlinear behavior of FG thin plates and shallow shells was studied by Woo and Meguid (2001). An analytical solution was provided for the large deflection of plates and shallow shells under mechanical loads and temperature. Shen (2002) investigated the nonlinear bending analysis of simply supported FG rectangular plates subjected to uniform or sinusoidal transverse loads and temperature. The perturbation technique was used by Huang and Shen (2004) to study the nonlinear vibration and dynamic response of FG plates in a thermal environment. Temperature-dependent material properties where considered. Qian et al. (2004) analyzed the static deformations and free and forced vibrations of a thick FG rectangular plate using a higher-order shear deformation plate theory and a meshless method. Shear deformable FG plates applied to nonlinear vibration analysis in a general state of non-uniform initial stress were studied by Chen (2005). Woo et al. (2006) obtained an analytical solution for the nonlinear free vibration of FG thin rectangular plates. Allahverdizadeh et al. (2008a, 2008b) developed a semi-analytical approach for nonlinear free and forced axisymmetric vibrations of FG thin circular plates under thermal effects. All the above studies are limited to plates with regular shapes.

Sector plates are commonly used in practical applications. Thus, the study of these plates becomes very important. Liew et al. (1995)

* Corresponding author. Tel./fax: +213 43 28 56 85.
E-mail address: belaliasidou@yahoo.fr (S.A. Belalia).

0997-7538/$ – see front matter © 2012 Elsevier Masson SAS. All rights reserved.
doi:10.1016/j.euromechsol.2012.01.004

Nomenclature

x, y	Cartesian coordinates
ξ, η	Non–dimensional coordinates
b	Radius
h	Thickness
ϕ	Sector angle
t	Time
u, v	In-plane displacements
w	Out-of-plane displacement
P_r^*	Shifted Legendre orthogonal polynomial of order r
n	Volume fraction exponent
r	Dimension of shape functions
φ_x	Rotation about x axis
φ_y	Rotation about y axis
p	Polynomial order
P	Material properties
ρ_m	Metal mass density
E_m	Metal elastic modulus
ν	Poisson's ratio
k	$=\pi^2/12$ (Shear correction factor)
U	Strain energy
T	Kinetic energy
$\overline{\mathbf{K}}$	In-plane stiffness matrix
$\mathbf{K}$	Out-of-plane linear stiffness matrix
$\breve{\mathbf{K}}$	Coupling stiffness matrix
$\hat{\mathbf{K}}$,$\tilde{\mathbf{K}}$	Nonlinear stiffness matrices
$\overline{\mathbf{M}}$	In-plane mass matrix
$\mathbf{M}$	Out-of-plane mass matrix
$\bar{\mathbf{q}}$	In-plane displacement vector
$\mathbf{q}$	Out-of-plane displacement vector
ξ_i, η_j	Abscissas for Gauss-Legendre quadrature
w_i, w_j	Weights for Gauss-Legendre quadrature
m	Number of quadrature points
$w_{\max}$	Maximum amplitude
ω	Natural frequency
Ω	$=\dfrac{\omega b^2}{h}\sqrt{\dfrac{12\rho_m(1-\nu^2)}{E_m}}$ (non-dimensional frequency parameter)

presented a literature survey of the vibration of plates of various shapes including sector plates. Liu and Liew (1999) analyzed the free vibration of moderately thick sectorial plates using Mindlin plate theory in conjunction with the differential quadrature method. Huang and Ho (2004) presented an analytical solution for the vibration of a polar orthotropic sector plate with simply supported radial edges. An exact solution was presented for the free vibration of a transversely isotropic sector plate by Jomehzadeh and Saidi (2009). For the linear vibration of FG sector plates, a few works exist in the literature. Nie and Zhong (2008) studied the free and forced vibration of FG annular sector plates with simply supported radial edges and arbitrary circular edges using a semi-analytical approach. Recently, Sahraee (2009) investigated the bending analysis of FG thick sector plates using Levinson and Mindlin plate theories. To the best of authors' knowledge, solutions for the nonlinear free vibration of FG Mindlin sector plates are not yet available in the literature.

The p-version of the finite element method also known as the hierarchical finite element method leads to a higher accuracy than the conventional finite element method with fewer degrees of freedom (Suri et al., 1995; Suri, 1996; Ribeiro and Petyt, 1999; Ribeiro, 2003). The first work which employed the p-version of the finite element method in conjunction with the harmonic balance method to the geometrically nonlinear free vibration of isotropic thin rectangular plates is attributed to Han and Petyt (1997). The resultant nonlinear equations were solved iteratively using the linearized updated mode technique. Leung and Zhu (2004) presented a trapezoidal p-element for the geometrically nonlinear free vibration of moderately thick clamped polygonal plates. Houmat (2008) developed a sector p-element and applied it to large amplitude free vibration analysis of shear deformable laminated composite annular sector plates. Houmat (2009) presented an elliptic sector p-element for the nonlinear free vibration analysis of shear deformable laminated composite annular elliptical plates. Belalia and Houmat (2010) developed a curved triangular p-element and applied it to nonlinear free vibration analysis of elliptic sector plates. The geometry of the element was described accurately using the blending function method (Gordon, 1971).

This paper presents a p-version of the finite element method based on a curved triangular p-element for the geometrically nonlinear free vibration analysis of FG Mindlin sector plates. This method is actually an h-p finite element procedure. In fact, the number of elements and the degree p of the interpolating polynomial may vary simultaneously. However, when dealing with the nonlinear free vibration of a sector plate, it is more convenient to model the domain of the plate using one p-element. In this way, the task of finding the position of the point of maximum amplitude will be simpler than if using several elements. The material properties of the constituents are graded in the thickness direction according to a power-law distribution. The blending function method is used to describe accurately the geometry of the sector plate. The shape functions are expressed in terms of the shifted Legendre orthogonal polynomials. Mindlin plate theory is employed. The Von Karman's assumptions are used in conjunction with the harmonic balance method to derive the equations of motion. The resultant nonlinear equations are solved iteratively using the linearized updated mode technique. The convergence and accuracy of the method are investigated. Comparisons are made with published results. The effects of sector angle, thickness, material type, and volume fraction exponent on the hardening behavior are studied.

2. Properties of functionally graded materials

An FG sector plate with uniform thickness h is made out of a graded mixture of ceramics and metal as shown in Fig. 1. The top layer ($z = h/2$) of the FG sector plate is a pure ceramic material and the bottom layer ($z = -h/2$) is a pure metallic material.

The material properties E,ν, and ρ of the FG sector plate are assumed to be graded in the thickness direction according to a power-law distribution which is expressed (Praveen and Reddy, 1998) as

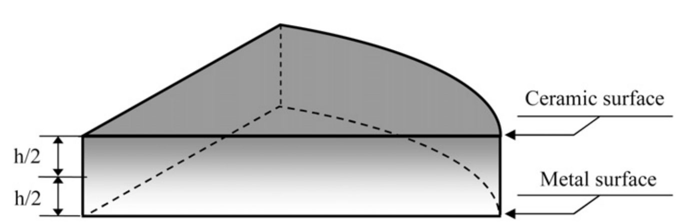

Fig. 1. The FG sector plate.