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# Predictors of fatigue damage accumulation in the neighborhood of small notches

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ABSTRACT

The effects of small notches on the predicted fatigue life of mechanical and structural components are examined. It is shown that the classical results of Neuber, Peterson and others can be interpreted to mean integral averages of stresses or strains over small volumes that depend on the solution of a problem of elasticity. This interpretation permits generalization of those results to cases that lie outside of the range of parameters considered in conventional mechanical design. Examples are presented.


## 1. Introduction

Improvements in the reliability of estimating fatigue life of structural and mechanical components exposed to cyclic loading continues to be a subject of investigation motivated by the desire to improve design, maintenance and certification decisions for high value assets such as rotorcraft, turbomachinery and the like. In this paper we will be concerned with the prediction of fatigue life of metallic parts.

Design rules for static and dynamic problems are usually written in terms of maximum stress and/or strain. By their precise definition, stress and strain are abstract mathematical objects defined for perfectly homogeneous continua. Applied to real materials, one must distinguish between stress (resp. strain) on the macroscopic scale from stress (resp. strain) on the microscopic scale. Stress (resp. strain) on the macroscopic scale should be understood as the average of stress (resp. strain) on the microscopic scale over a representative volume element (RVE). The physical processes that lead to fatigue damage occur on the microscopic scale. These are highly complex, irreversible processes. It is assumed that these processes can be correlated with macroscopic stress and strain cycles by means of phenomenological models, called predictors. We will be concerned with stress and strain on the macroscopic scale and the formulation and evaluation of predictors intended for the generalization of the results of fatigue coupon tests.

The maximum stress calculated on the basis of classical engineering formulas for axial loading, bending and torsion is generally not the same as stresses corresponding to models based on the theory of elasticity. This is because the engineering formulas are based on certain simplifying assumptions concerning the displacement field. Geometric stress concentration factors, usually denoted by $K_t$, are commonly used to reconcile this difference in engineering practice.

Calibration data, represented in the form of S-N curves, are used for estimating the fatigue lives of mechanical components. S-N curves correlate average stresses with cycles to failure. The failure process is a combination of crack initiation and crack propagation. The size of a nucleated fatigue crack that can be detected in fatigue experiments through decrease of stiffness of the test article is approximately 0.5 mm [1]. In most cases only the total number of cycles to failure is recorded, however.

It was observed by early investigators that using the maximum stress in the neighborhood of notches would underestimate the average number of cycles to failure. For this reason the maximum stress $\sigma_{\max}$ was replaced by an effective stress $\sigma_{\text{eff}}$ (where $\sigma_{\text{eff}} \leqslant \sigma_{\max}$) when making estimates of fatigue life on the basis of S-N curves. Because the primary interest of the investigators was to estimate the fatigue life of notched and filleted mechanical and structural components, they assumed that in the absence of residual stresses the ratio $\sigma_{\text{eff}}/\sigma_{\max}$ depends on $K_t$, the notch radius

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$r$ and an empirical material-dependent constant. See, for example, Eq. (1.55) and Fig. 1.31 in [2]. This approach has been used successfully for the problem class for which it was intended.

We are interested in the questions of (a) how to estimate fatigue damage in cases for which the classical approaches, such as those of Neuber [3] and Peterson [2], are not applicable, and (b) how alternative generalizations of uniaxial calibration data should be evaluated and ranked against available experimental information using the concepts and procedures of verification, validation and uncertainty quantification.

This paper is organized as follows: The properties that predictors of damage accumulation must satisfy are described and a generic formulation of a class of predictors is presented in Section 2. It is shown in Section 3 that $\sigma_{\text{eff}}$ computed using the notch sensitivity index defined in [2] is equivalent to computing the average stress over a domain which depends on the solution. The interpretation and generalization of fatigue data published in a series of NACA and NASA reports, calibration of a predictor characterized by two model form parameters and procedures for ranking mathematical models are discussed in Section 4. A summary is presented in Section 5.

## 2. Predictors of damage accumulation

Predictors of damage accumulation are phenomenological models constructed for the purpose of generalizing sets of experimental data obtained in fatigue tests of mechanical or structural components subjected to cyclic loading. Predictors are scalars that depend on the solution of a problem of elasticity, or more generally, a problem of continuum mechanics. Various predictors have been proposed.

For example, one of the predictors defined in [4] is $\sqrt{\sigma_{\max}\epsilon_a E}$ where $\sigma_{\max}$ is the maximum normal stress, $\epsilon_a$ is the strain amplitude and $E$ is the modulus of elasticity. Another predictor, proposed for high cycle fatigue without pre-strain, is $\sqrt{\sigma_{\max}\sigma_a}$ where $\sigma_a$ is the stress amplitude:

$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2} \tag{1}$$

hence

$$\sqrt{\sigma_{\max}\sigma_a} = \sigma_{\max}\left(\frac{1-R}{2}\right)^{1/2} \tag{2}$$

where $R = \sigma_{\min}/\sigma_{\max}$ is the stress ratio. Data presented in [4] convincingly demonstrate that this predictor, plotted against the number of cycles to failure on log–log scale, permit reduction of data from many tests with different cycle ratios to a single S-N curve, allowing for small statistical variations.

Calibration curves are typically obtained under uniaxial stress conditions, less frequently under biaxial stress conditions. To permit generalization, predictors must be interpreted to cover triaxial stress conditions. Several plausible generalizations of uniaxial and biaxial fatigue data are possible. For example, assuming constant amplitude cyclic loading, the predictor represented by Eq. (2) can be understood to mean

$$\sqrt{\sigma_{\max}\sigma_a} = P(I_1) \equiv \sqrt{(I_1)_{\max}(I_1)_a} \tag{3}$$

where $I_1 > 0$ is the first stress invariant and $(I_1)_a$ is the amplitude of $I_1$ in the location of $(I_1)_{\max}$. Furthermore, $R = (I_1)_{\min}/(I_1)_{\max}$.

Another possible generalization is that $\sigma_{\max}$ is the maximum value of the von Mises stress $\bar{\sigma}$ defined by

$$\bar{\sigma} = \sqrt{\frac{1}{2}\left((\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right)} \tag{4}$$

where $\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3$ are the principal stresses. The predictor represented by Eq. (2) can be understood to mean

$$\sqrt{\sigma_{\max}\sigma_a} = P(\bar{\sigma}) \equiv \sqrt{\bar{\sigma}_{\max}\bar{\sigma}_a} \tag{5}$$

and $R = \bar{\sigma}_{\min}/\bar{\sigma}_{\max}$.

While the calibration curve (S-N curve) for both predictors is the same, the two generalizations are contradictory: The generalization represented by Eq. (3) states that damage accumulation can be correlated with the first invariant of the stress tensor $I_1$ whereas the generalization represented by Eq. (5) states that damage accumulation is independent of $I_1$.

It is possible to construct predictors as convex combinations of the two generalizations represented by Eqs. (3) and (5):

$$P(I_1, \bar{\sigma}, \alpha) = \alpha I_1 + (1-\alpha)\bar{\sigma}, \quad 0 \leqslant \alpha \leqslant 1 \tag{6}$$

in which case the two generalizations are complementary for $\alpha \neq 0, 1$. The underlying assumption is that both $I_1$ and $\bar{\sigma}$ contribute to the formation and initial propagation of cracks. The parameter $\alpha$ can be inferred from experimental data. This is discussed in Section 4.5. Some models proposed for the generalization of the endurance limit from uniaxial to triaxial stress conditions are also based on a combination of $I_1$ and $\bar{\sigma}$. See for example [5–7].

**Remark.** Criteria proposed to predict failure in high cycle fatigue have been classified in two broad categories: *local* criteria are criteria based on the maximum stress or strain and *non-local* criteria based on some functional of the stress or strain field. Examples of local criteria are the classical criteria of Neuber [3] and Peterson [2]. An example of non-local criteria is the theory of critical distances [8]. There are many criteria within each category. The distinction between the two categories is blurred by the fact that the classical criteria can be viewed as applications of non-local criteria [9]. A summary of several local and non-local criteria is presented in [10,1].

### 2.1. Basic properties

Several predictors of damage accumulation have been proposed for the purpose of generalizing fatigue data. A predictor is a phenomenological model and as such has a range of validity. Proper definition of a predictor must include an indication of the range of parameters for which its validity and usefulness is supported by experimental evidence. Predictors are scalars that satisfy the following conditions:

1. A predictor corresponding to the exact solution of a mathematical model used for the determination of stress and strain fields must be a finite number.
2. A predictor must be independent of the choice of coordinate system.
3. Small changes in the input data must not produce large changes in the predictor.
4. It must be possible to determine the critical value of a predictor from the outcome of coupon tests. Therefore the predicted event must be such that it can be observed directly or inferred from observations.

The mathematical models used in estimating fatigue lives are typically based on the theory of elasticity, more generally on small strain continuum theory. Item 2 is known as the condition of objectivity. A tensor function satisfies the condition of objectivity if for some function $\mathcal{F} : \mathbb{R}^3 \to \mathbb{R}$ we have $\mathcal{F} = \mathcal{F}(\mathcal{I}, \mathcal{T})$ where $\mathcal{I}$ is some linear combination of the invariants of the stress and/or strain tensor and $\mathcal{T}$ is the temperature.