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Size-dependent energy release rate formulation of notched beams based on a modified couple stress theory

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ABSTRACT

The modified couple stress theory is employed in this paper in order to formulate the size-dependent strain energy release rate of Euler–Bernoulli and Timoshenko notched beams. As a case study, the normalized energy release rate of the aforementioned beams has been developed for three-point bending as a function of the ratio of material length scale parameter to the beam depth. The results of the current model are compared to the experimental data. The good agreement between the present results and the experimental values indicates that the current model can be successfully applied to evaluate size dependent energy release rate of structures.

1. Introduction

Fracture toughness, i.e. resistance of a structure to fracture in presence of flaws such as cracks, voids, inclusions, is one of the most important design parameter in structural analysis. one of the main issues in fracture mechanics is determination of the strain energy release rate acting as the crack extension force from which the stress intensity factors can be calculated via Irwin's relation [1].

In order to determine the energy release rate or stress intensity factor in practical problems, the researchers have employed some methods such as experimental [2,3], numerical [4,5] and analytical [6,7] methods. Analytical methods for estimation of stress intensity factor in the linear elastic fracture mechanics approach are considered as [8]: the direct methods, the energy based methods, the singularity function methods, the superposition method and the boundary integral equation methods.

A remarkably simple method for close approximation of energy release rate, G in notched beams was discovered by Kienzler and Herrmann [9] and Herrmann and Sosa [10]. The method was derived from a certain unproven hypothesis (postulate) regarding the energy release when the thickness of the fracture band is increased [11].

Due to the vast applications of Micro-Electro-Mechanical-Systems (MEMS) such as microresonators [12], micropumps [13] and Atomic Force Microscopes (AFM) [14], many researchers have been studying the behavior of the micromechanical components used in MEMS. Since these components usually work at high operating frequencies, they are subjected to very high numbers of fatigue cycles during their life. Hence, investigation of fatigue mechanism in MEMS seems to be crucial. To that end, the calculation of strain energy release rate for MEMS mechanical components would be helpful.

The experimental observations indicate that the mechanical behavior of micro/sub-micro-scale structures is size-dependent that cannot be justified by the classical continuum mechanics [15–17]. Since the attempts of the classical continuum theories has been in vain to predict and justify the small scale effect leading to size-dependency in micromechanical ele-

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Nomenclature

Latin characters

a	crack length
A	cross-section area
b	material load
$\bar{b}$	beam width
B	material force
d_a	concrete aggregate size
E	elastic modulus
f_C	compressive strength
g	material microstructural length
G	strain energy release rate
G_f	fracture energy
G_{IC}	size-dependent critical energy release rate
h	beam height
I	area moment of inertia
l	material length scale parameter
L	half of the beam length
m	external couple per unit length
m_{ij}	components of the deviatoric part of the couple stress tensor
M	bending moment
P	three-point bending load
q	lateral force per unit length
Q	transverse shear force
s	beam span
$\mathbf{u}$	displacement vector
V	potential of the external loads
w	lateral deflection
W	strain energy density

Greek characters

α	ratio of the length scale parameters to the beam thickness
δ_{ij}	kronecker delta function
ε_{ij}	components of the symmetric part of strain tensor
ζ	ratio of the elastic modulus to the shear modulus
η	ratio of the beam length to the beam thickness
$\boldsymbol{\theta}$	rotation vector
μ	shear modulus
v	Poisson ratio
π	total potential energy
σ_{ij}	components of the symmetric part of stress tensor
χ_{ij}	components of the symmetric part of the curvature tensor
ψ	rotation angle of cross-sections

Subscripts

E	Euler–Bernoulli beam theory
T	Timoshenko beam theory

Superscripts

cl	classical beam theory
C	classical part
$N \cdot C$	non-classical part
T	transpose

ments, some non-classical continuum theories such as the non-local theory, the strain gradient theory and the couple stress theory have been emerged and developed during past years. By considering higher-order stresses known as the couple stress in addition to the classical stresses, the couple stress theory has been developed [18–20]. In this non-classical theory, there exists two material length scale parameter beside the two well-known Lamé constants which enables the theory to capture