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Three-dimensional *T*-stresses for three-point-bend specimens with large thickness variation

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ABSTRACT

Three-point-bend (3 PB) test specimens are useful for the systematic investigation of the influence of statistical and constraint loss size effects on the cleavage fracture toughness of a material in the ductile-to-brittle transition temperature range. Because the in- and out-of-plane elastic *T*-stresses (T_{11} and T_{33}) are a measure of the crack-tip constraint and even the in-plane T_{11} exhibits three-dimensional (3D) effects, the 3D *T*-stresses solutions were obtained by running finite element analyses (FEA) for 3 PB specimens with wide ranges of the crack depth-to-width ratio ($a/W = 0.2–0.8$) and the specimen thickness-to-width ratio ($B/W = 0.1–40$). The results show that the 3D T_{11} at the specimen mid-plane tended to deviate from the 2D T_{11} as B/W increased, with the deviation saturating for $B/W \geqslant 2$. The mid-plane T_{33} increased with B/W and was close to the plane strain value νT_{11} for $B/W \geqslant 2$.

1. Introduction

Three-point-bend (3 PB) test specimens are useful for the systematic investigation of the statistical and constraint loss size effects on the cleavage fracture toughness of a material in the ductile-to-brittle transition temperature range [1,2]. Because the in-plane and out-of-plane *T*-stresses (T_{11} and T_{33}) are a measure of the crack-tip constraint and even the in-plane T_{11} exhibits three-dimensional (3D) effects [2–4], the 3D *T*-stresses solutions were obtained by running finite element analyses (FEA) for 3 PB specimens with wide ranges of the crack depth-to-width ratio ($a/W = 0.2–0.8$) and the specimen thickness-to-width ratio ($B/W = 0.1–40$). The 2D T_{11} solutions have been provided for 3 PB specimen in many numerical studies [5–10].

The results show that the 3D T_{11} at the specimen mid-plane tended to deviate from the 2D T_{11} as B/W increased, with the deviation saturating for $B/W \geqslant 2$. The mid-plane 3D T_{11} for $B/W = 0.1$ and 40 was high as 54% when $a/W = 0.2$, suggesting that 3D effects should be properly considered for cases of short crack length, especially when T_{11} is negative. The mid-plane T_{33} increased with B/W and was close to the plane strain value νT_{11} for $B/W \geqslant 2$.

2. *T*-stress

In an isotropic linear elastic body containing a crack subjected to symmetric (mode I) loading, the Williams series expansion [11] of the 3D stress components near the crack tip field can be written as [3]

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Nomenclature

B	specimen thickness
E	Young's modulus
F	unit magnitude (see Eq. (2))
I	interaction integral
K_I	local mode I stress intensity factor (SIF)
K_0	2D SIF for elastic analysis
R_s	crack tube radius
S	support span for 3PB specimen
T_{11}, T_{33}	T-stresses
W	specimen width
a	crack length
r, θ	in-plane polar coordinates
x_j	crack-tip local coordinates (j = 1, 2, 3)
Δl	singular element size
β_{11}, β_{33}	normalized T-stresses
ε_{33}	out-of-plane strain
ν	Poisson's ratio
σ_{ij}	stress components (i, j = 1, 2, 3)

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{Bmatrix} = \frac{K_I}{\sqrt{2\pi r}} \begin{Bmatrix} \cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \\ \cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \\ 2\nu\cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} T_{11} \\ 0 \\ T_{33} \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{1}$$

where r and θ are the in-plane polar coordinates of the plane normal to the crack front shown in Fig. 1, K_I is the local mode I stress intensity factor (SIF) and ν is Poisson's ratio. Here, x_1 is the direction formed by the intersection of the plane normal to the crack front and the plane tangential to the crack plane. T_{11} and T_{33} are the amplitudes of the second-order terms in the three-dimensional series expansions of the crack front stress field in the x_1 and x_3 directions, respectively.

Different methods have been applied to compute the elastic T-stress for test specimens, as summarized by Sherry et al. [10]. In this study, an efficient finite element method developed by Nakamura and Parks [3] based on an interaction integral was used to determine the elastic T-stresses.

The crack tip T_{11}-stress on the crack front is related to the interaction integral by

$$T_{11} = \frac{E}{1-\nu^2}\left\{\frac{I}{F} + \nu\varepsilon_{33}\right\} \tag{2}$$

where E is Young's modulus, ν is Poisson's ratio and ε_{33} identifies the out-of-plane strain at the crack tip in the direction tangential to the crack front. I represents the interaction integral, and F indicates the unit magnitude ($F = 1$).

Once the T_{11}-stress is obtained, the T_{33}-stress can be obtained using the following relationship:

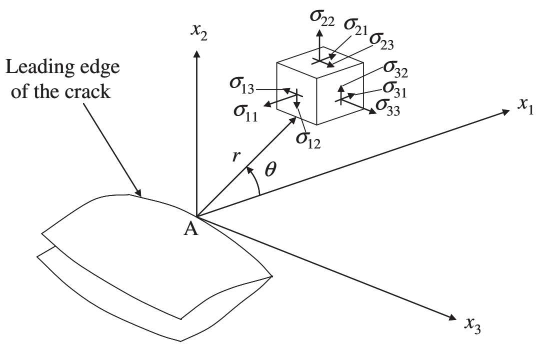

Fig. 1. Three-dimensional coordinate system for the region along the crack front.