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Interaction laws and mechanisms of the multiple unequal cracks on the two coaxial interfaces in a tri-layered multiferroic semi-cylinder

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ABSTRACT

Mode III fracture analysis is performed on the multiple unequal interfacial cracks in a tri-layered multiferroic semi-cylinder by the methods of dislocation simulation, Green's function and Cauchy singular integral equation. Based on the parametric studies on the numerical results of stress intensity factor (SIF), the interaction laws of the cracks, especially the local oscillation phenomenon of the SIF curves, are revealed, and the boundary effect, the shielding effect and the interference effect are employed to interpret the underlying mechanisms. The obtained results can provide theoretical references to the anti-fracture optimal design of this kind of smart structures.

1. Introduction

Multiferroic composites consisting of alternate ferroelectric (FE) and ferromagnetic (FM) layers have piezoelectric, piezomagnetic and magneto-electric effects simultaneously at room temperature [1,2]. Therefore, they are ideal candidates for making small and multifunctional intelligent devices [3]. In engineering, multiferroic devices are generally designed as layered structures such as laminated cylinders or plates [4]. For these structures, interfaces are significant for loading transmission but meanwhile subjected to damages such as cracking [5,6]. Interface debonding occasionally occurs when multiferroic composites are applied by mechanical, electrical and/or magnetical loading. Therefore, interface fracture mechanics is a key fundamental research in this field with wide applications in optimal designs of this kind of smart structures.

With the wider and wider applications of smart devices, interfacial fracture analysis on FE/FM composites has become a hotspot recently. In existing papers, FE/FM composites are generally simplified as bi-layered structures that only have a single interface [7–12]. However, practical multiferroic devices are multi-layered structures, having several and even multiple parallel interfaces. When two neighboring interfaces are debonded simultaneously, their interfacial cracks may have interactions, resulting in a fracture behavior more complex than that in the case of a single interface. Therefore, investigation on the interaction laws between cracks on two neighboring and parallel interfaces is significant for the anti-fracture optimal designs of this kind of composites. In the present work, the theoretical model is established for a tri-layered semi-cylinder of alternate FE and FM ceramics. The problem of simultaneous cracking of the two coaxial interfaces is analyzed, and the associated interaction laws are revealed. The underlying mechanisms are explained by using the effects of stiff boundary, free boundary, shielding and interference.

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Nomenclature

w, φ and ϕ mechanical displacement, magnetic potential and electric potential
τ, B and D stress, magnetic induction and electric displacement
$\boldsymbol{\tau}$ and $\boldsymbol{w}$ generalized stress and generalized displacement
c_{44}, h_{15}, μ_{11}, e_{15} and ε_{11} elastic constant, piezomagnetic coefficient, magnetic permeability, piezoelectric coefficient and dielectric coefficient
h_1, h_2 and h_3 layer thickness
$\left(\alpha_i^{(\mathrm{I})}, \beta_i^{(\mathrm{I})}\right)$ and $\left(\alpha_j^{(\mathrm{II})}, \beta_j^{(\mathrm{II})}\right)$, $(i = 1, 2, \ldots, n_{\mathrm{I}};\ j = 1, 2, \ldots, n_{\mathrm{II}})$ angular intervals of the cracks
$\boldsymbol{A}$ and $\boldsymbol{C}$ undetermined coefficient function vectors
$g_i^{(\mathrm{I})}(\theta)$ and $g_j^{(\mathrm{II})}(\theta)$, $(i = 1, 2, \ldots, n_{\mathrm{I}};\ j = 1, 2, \ldots, n_{\mathrm{II}})$ dislocation density functions
γ_{I} and γ_{II} angular positions of the two dislocation points
$Q_i(n)$, $(i = 1, 2, \ldots, 10)$ known functions
k_{e} and k_{m} piezoelectric and piezomagnetic stiffening factors
q_1, q_2 and λ dimensionless constants
G_{rz} Green's function
$-\tau_1$ and $-\tau_2$ equivalent tractions on the crack surfaces
$\widetilde{R}_{ba}(\tilde{\zeta}_t, \tilde{\xi}_r), \widetilde{R}_{bb}(\tilde{\zeta}_t, \tilde{\xi}_r), \widetilde{R}_{aa}(\tilde{\zeta}_t, \tilde{\xi}_r)$ and $\widetilde{R}_{ab}(\tilde{\zeta}_t, \tilde{\xi}_r)$ dimensionless kernel functions
$K_{\alpha i}^{(\mathrm{I})}, K_{\beta i}^{(\mathrm{I})}, K_{\alpha j}^{(\mathrm{II})}$ and $K_{\beta j}^{(\mathrm{II})}$ stress intensity factors

Multiple cracks might occur in FE and/or FM devices after their long-time service in severe environments. The interaction among them will make the fracture behavior much more complex. Therefore, the interaction problems of multiple cracks have formed an active topic in fracture mechanics of FE and FM ceramics. Han and Chen [13] proposed the pseudo-traction-electric displacement method to solve the interaction problem of multiple parallel cracks in transversely isotropic piezoelectric ceramics, and surveyed the role that the electric displacement loading plays in the interaction problem. Zeng and Rajapakse [14] investigated the interaction between a semi-infinite main crack with an arbitrarily located and oriented microcrack in a PE plane and revealed the shielding and amplification effects therein. Using a dielectric crack model, Wang and Jiang [15] analyzed the effective electro-elastic property of PE media weakened by parallel cracks. Tian and Gabbert [16] studied the interaction of multiple arbitrarily oriented and distributed cracks in magnetoelectroelastic materials by the method of pseudo-traction–electric-displacement–magnetic-induction. Li and Lee [17] solved the problem of periodic distribution of slant mode-III cracks in a piezoelectric material by simulating the cracks as pile-ups of screw dislocations. The problems of two or four parallel mode-I cracks in piezoelectric materials were analyzed under the permeable-crack assumption by Zhou and Wu [18] and under the limited-permeable-crack assumption by Zhou et al. [19]. Zhang et al. [20] discussed the effects of the lengths and spacing of cracks and the material parameters on the stress intensity factors of four parallel non-symmetric permeable mode-III cracks in a functionally graded piezoelectric material. Li and Ding [21] surveyed the problem of a periodic array of cracks in a functionally graded piezoelectric strip bonded to a different functionally graded piezoelectric plane.

A common characteristic of these papers is that they are concerned with multiple cracks in an infinite plane. However, practical multiferroic composites are generally laminated plates or cylinders. The interaction problems of the multiple cracks in laminated FE/FM structures of finite dimension have been scarcely investigated. Our research team performed theoretical analysis on the mixed mode fracture of a FE/FM bi-material with two un-coaxial cracks parallel to the interface and each in a layer, and revealed the interaction laws of the two cracks [12]. In the present paper, we continue to investigate the interactions between two groups of anti-plane cracks in a tri-layered multiferroic semi-cylinder by the methods of continuously distributed dislocations, Green's functions and Cauchy singular integral equations. The original aspects of this article can be stated as follows: (a) it considers multiple interface cracks in a finite domain as opposed to existing articles where multiple cracks have been investigated in infinite planes and (b) existing studies in this field generally dealt with bi-layers whereas the current article extends the analysis to tri-layers.

2. Problem formulation

Illustrated in Fig. 1 is the cross section of a tri-layered semi-cylinder consisting of the outer and inner FE layers and an intermediate FM layer. For the convenience of description, the inner and outer interfaces are called interface I and II, whose radii are r_1 and r_2. The radii of the inner and outer surfaces are denoted by r_0 and r_3 accordingly. There are two groups of multiple unequal interfacial cracks, each group on an interface. The angular intervals occupied by the cracks are denoted