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Nonlocal elasticity based vibration of initially pre-stressed coupled nanobeam systems

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ABSTRACT

Vibration analyses of coupled nanobeam system under initial compressive pre-stressed condition are presented. An elastically connected double-nanobeam-system is considered. Expressions for bending-vibration of pre-stressed double-nanobeam-system are formulated using Eringen's nonlocal elasticity model. An analytical method is proposed to obtain natural frequencies of the nonlocal double-nanobeam-system (NDNBS). Nano-scale effects and coupling spring effects in (i) in-phase type, (ii) out-of-phase type vibration; and (ii) vibration with one nanobeam fixed are examined. Scale effects in higher natural frequencies of NDNBS are also highlighted in this manuscript. Results reveal the difference (quantitatively) by which the pre-load affects the nonlocal frequency in the in-phase type and out-of-phase type vibrations mode of NDNBS.

1. Introduction

A beam is a simple model of one-dimensional continuous system (Timoshenko, 1953). Its importance in various engineering fields is well appreciated (Jennings, 2004). Beam-type structures are widely used in many branches of modern civil, mechanical and aerospace engineering. Recently, it is being extensively utilized as nanostructure components (Harik and Salas, 2003) for nanoelectromechanical (NEMS) and microelectromechanical systems (MEMS). Being important from the theoretical and engineering points of view, the dynamic problems involving one-dimensional continuous beam have drawn great deal of attention over the past few decades.

An important technological extension of the concept of the single beam is that of the complex coupled-beam-systems. One such simple coupled beam system is the double-beam-system. The double-beam-system is a continuous system consisting of two one-dimensional beams joined by an elastic medium represented by distributed vertical springs. Employing beam theories, several important works on vibration and buckling of elastically connected double-beam systems are reported. Vu et al. (2000) studied the vibration of homogenous double-beam system subjected to harmonic excitation. Erol and Gurgoz (2004) extended the analysis of Vu et al. (2000) to axially vibrating double-rod system coupled by translational springs and dampers. Oniszczuk (2000a) studied the free vibrations of two parallel simply supported beams continuously joined by a Winkler elastic layer. Undamped forced transverse vibrations of an elastically connected simply supported double-beam system were analysed. Free and forced vibration of double-string complex system was also investigated by Oniszczuk (2000b, 2000c). Hilal (2006) investigated the dynamic response of a double Euler–Bernoulli beam due to moving constant load. The effects of the speed of the moving load, the damping and the elasticity of the coupling viscoelastic layer on the dynamic responses of the beam system were presented. Vibration analysis of double-beam systems interconnected only at discrete points was reported by Hamada et al. (1983) and Gurgoz and Erol (2004). Buckling and the effect of a compressive load on the free and forced vibration on double-beam systems were reported by Zhang et al. (2008a, 2008b). Kelly and Srinivas (2009) carried out vibrations of elastically connected stretched beam systems. Analyses of double-beam systems by numerical techniques were also reported. Rosa and Lippiello (2007) presented non-classical boundary conditions and differential quadrature method for vibrating double-beams. Li and Hua (2007) presented spectral finite element analysis of elastically connected double-beam systems.

From the above discussion, it can be observed that the vibration theory of double-beam systems is well developed and studied in details. However, there are only few contributions dealing with the vibrations of beam-systems which are scale-dependent. Scale-dependent beams structures are those fabricated from nanomaterials. The nanomaterials are future generation engineering materials and have stimulated the interest of the scientific researcher's communities in physics, chemistry, biomedical and

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engineering. These nanomaterials have special properties resulting from their nanoscale dimensions. Common examples of materials that exhibit interesting properties on the nanoscale include nanoparticles, nanowires and nanotubes (viz. carbon nanotubes, ZnO nanotubes). These nanomaterials have promising mechanical (tensile strength), chemical, electrical, optical and electronic properties (Dai et al., 1996; Bachtold et al., 2001; Kim and Lieber, 1999). Because of many desirable properties, nanomaterials are perceived to be the components for various nanoelectromechanical systems (NEMS) and nanocomposites. Structural beams fabricated from nanomaterials and of nanometre dimension are referred as nanobeams.

The understanding of dynamics of single-nanobeam (carbon nanotubes, nanowires) is important. The vibration characteristics of nanobeams can be employed for NEMS/MEMS applications (Pugno et al., 2005; Ke et al., 2005a, 2005b). Parallel to vibration of single-nanobeam, the study of vibrating multiple-nanobeam-system is also relevant for nanosensors and nanoresonators applications. The recent development of nano-optomechanical systems (NOMS) necessitates the use of vibrating double-nanobeam-systems.

The employment of double-nanobeam-systems in NOMS has been reported by various researchers. Frank et al. (2010) presented a dynamically reconfigurable photonic crystal nanobeam cavity. There work involved two closely situated parallel vibrating clamped double-nanobeam-systems. Eichenfield et al. (2009) described the design, fabrication, and measurement of a cavity nano-optomechanical system (NOMS). The NOMS consisting of two closely separated coupled nanobeams. The researchers fabricated the low dimension double-beam-system by depositing stoichiometric silicon nitride using low-pressure-chemical-vapour-deposition on a silicon wafer. Deotare et al. (2009) studied the coupled photonic crystal nanobeam cavities consisting of two parallel suspended nanobeams separated by a small gap. The use of vibration properties in double-nanobeam-system has also been reported by Lin et al. (2010). The authors studied the coherent mixing of mechanical excitations in nano-optomechanical structures. Most of the works reported here are experimental works.

It is understood that controlling every parameter in experiments at nanoscale is difficult. Further, since molecular dynamics simulations are computationally expensive, analysis of nanostructures had been carried out by classical continuum theory. Extensive research over the past decade has shown that classical continuum models (Timoshenko, 1974) are able to predict the performance of 'large' nanostructures reasonably well. Classical continuum models are scale-free theory and it lacks the accountability of the effects arising from the size-effects. Experimental (Ruud et al., 1994; Wong et al., 1997; Sorop and Jongh, 2007; Kasuya et al., 1997; Juhasz et al., 2004) and atomistic simulations (Chowdhury et al., 2010) have evidenced a significant 'size-effect' in the mechanical properties when the dimensions of the nanostructures become 'small'. Size-effects are related to atoms and molecules that constitute the materials. The application of classical continuum models thus may be questionable in the analysis of 'smaller' nanostructures. Therefore, recently there have been research efforts to bring in the scale effects within the formulation by amending the traditional classical continuum mechanics. One widely used size-dependant theory is the nonlocal elasticity theory pioneered by Eringen (Eringen, 1972, 1983, 2002). Nonlocal elasticity accounts for the small-scale effects arising at the nanoscale level. Recent literature shows that the theory of nonlocal elasticity is being increasingly used (Peddieson et al., 2003; Sudak, 2003; Wang, 2005; Wang et al., 2006; Reddy, 2007; Wang and Wang, 2007; Wang and Varadan, 2007; Lu, 2007; Hu et al., 2008; Heireche et al., 2008; Reddy and Pang, 2008; Artan and Tepe, 2008; Sun and Liu, 2008; Aydogdu, 2009; Murmu and Pradhan, 2009a, 2009b; Pradhan and Murmu, 2010; Murmu and Adhikari, 2010a, 2012, 2011a, 2011b; Hao et al., 2010; Shen, 2010; Xiang et al., 2010; Murmu et al., 2011) for reliable and quick analysis of nanostructures viz. nanobeams, nanoplates, nanorings, carbon nanotubes, graphenes, nanoswitches and microtubules. For double-nanobeam-system, Murmu and Adhikari (2010b) studied the nonlocal effects in the longitudinal vibration of double-nanorod systems. Further using nonlocal elasticity Murmu and Adhikari (2010c) have proposed nonlocal transverse vibration analysis of coupled double-nanobeam-systems. The nonlocal elasticity has also potential in application in wide areas such as nanomaterials with defects (Pugno and Ruoff, 2004; Pugno, 2006a, 2006b).

In the nonlocal elasticity theory, the small-scale effects are captured by assuming that the stress at a point is a function of the strains at all points in the domain (Eringen, 1983). This is unlike classical elasticity theory. Nonlocal theory considers long-range inter-atomic interaction and yields results dependent on the size of a body. Some drawbacks of the classical continuum theory could be efficiently avoided and the size-dependent phenomena can be reasonably explained by the nonlocal elasticity theory. The majority of the existing works on nonlocal elasticity are pertaining to the free transverse vibration of single nanobeams. Though the mechanical studies of nanobeams may include vibration of multiple-walled nanotubes, the study of discrete multiple-nanobeam-system is particularly limited in literature.

Further it is observed that during the fabrication of nanostructures, the residual stresses can be developed within the structures. This initial residual stresses could significantly modify the mechanical and electrical properties of MEMS or NEMS devices. Strains are usually developed during the material growth and temperature relaxation. For a suspended structure, this process-induced strain may cause the axial residual stress within the structure. This calls for a deep understanding of its influence on the performance of the devices for the optimum design.

Therefore, based on the above discussion there is a strong encouragement to gain an understanding of the entire subject of vibration of complex-nanobeam-system and the mathematical modelling of such phenomena. In this paper an investigation is carried out to understand the small-scale effects in the free bending-vibration of nonlocal double-nanobeam-system (NDNBS) subjected to initial compressive pre-stressed load. The two nanobeams are subjected to initially pre-stress compressive loads. Initial pre-stressed compressive load may arise due to fabrication on nanobeam (Carr and Wybourne, 2003) or due to external applied compressive loads. Further, this paper presents a unique yet simple method of obtaining the exact solution for free vibration of double-nanobeam system. Equations for free bending-vibration of a pre-stressed double-nanobeam-system (NDNBS) are formulated within the framework of Eringen's nonlocal elasticity. The two nanobeams are assumed to be attached by distributed vertical transverse springs. These springs may represent the stiffness of an enclosed elastic medium, forces due to nano-optomechanical effects (Eichenfield et al., 2009; Deotare et al., 2009; Lin et al., 2010) or Vander Waals forces. An exact analytical method is proposed for solving the nonlocal frequencies of transversely vibrating NDNBS. The simplification in the computation is achieved based on the change of variables to decouple the set of two fourth-order partial differential equations. It is assumed that the two nanobeams in the NDNBS are identical, and the boundary conditions on the same side of the system are the same. Simply-supported boundary conditions are employed in this study. Explicit expressions for the natural frequencies of NDNBS are derived. The results are obtained for various vibration-phases of the NDNBS. The vibration phases include in-phase (synchronous) and out-of-phase (asynchronous) modes of vibration. The effects of (i) axial pre-stressed load, (ii) nonlocal parameter or scale coefficient, (iii) stiffness of the springs and (iv) the higher modes, on the frequency of the NDNBS are discussed.